

Strengthening Teaching Competences
in Higher Education
in Natural and Mathematical Sciences



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**Blended learning
Mathematics contents in
dynamic geometry
environment**

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Content:

- **VECTORS IN DINAMIC ENVIRONMENT**

- **DEFINITE INTEGRAL IN DINAMIC ENVIRONMENT**



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QUESTIONS:

How is blended learning created in a dynamic learning environment?

How can the surface area and the body volume in a vector dynamic learning environment be determined?

How can the lateral area and the volume of solid of revolution be determined by using a definite integral in a dynamic learning environment?



VECTORS IN DINAMIC ENVIRONMENT

Blended learning is created by the integration of new information and communication technologies, ICT, into the teaching process.

In the late 1990s and early 21st centuries, blended learning developed intensively with the advent of the Internet and the World Wide Web. But during the Covid 9 crisis it became the only learning method in the world.

By the term blended learning we will understand: distance learning, E-learning, as well as face-to-face learning.

The basic characteristics of distance learning are the physical distance between teacher and student

VECTORS IN DINAMIC ENVIRONMENT

1slika.ggb

Datoteka Uređivanje Prikaz Opcije Alati Prozor Pomoć

Algebarski prikaz Površina za crtanje Površina za crtanje 3D Površina za crtanje 2

- $h = 1$
- tekst1 = "M(a
- tekst2 = "M(2
- $H = (0, 0)$
- $b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
- $c = \begin{pmatrix} 2.1 \\ 0 \\ 0 \end{pmatrix}$
- $d = \begin{pmatrix} 2.1 \\ 2.1 \\ 0 \end{pmatrix}$
- $e = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$
- $i = \begin{pmatrix} 0 \\ 2.1 \\ \hat{} \end{pmatrix}$

$a_x = 2.1$

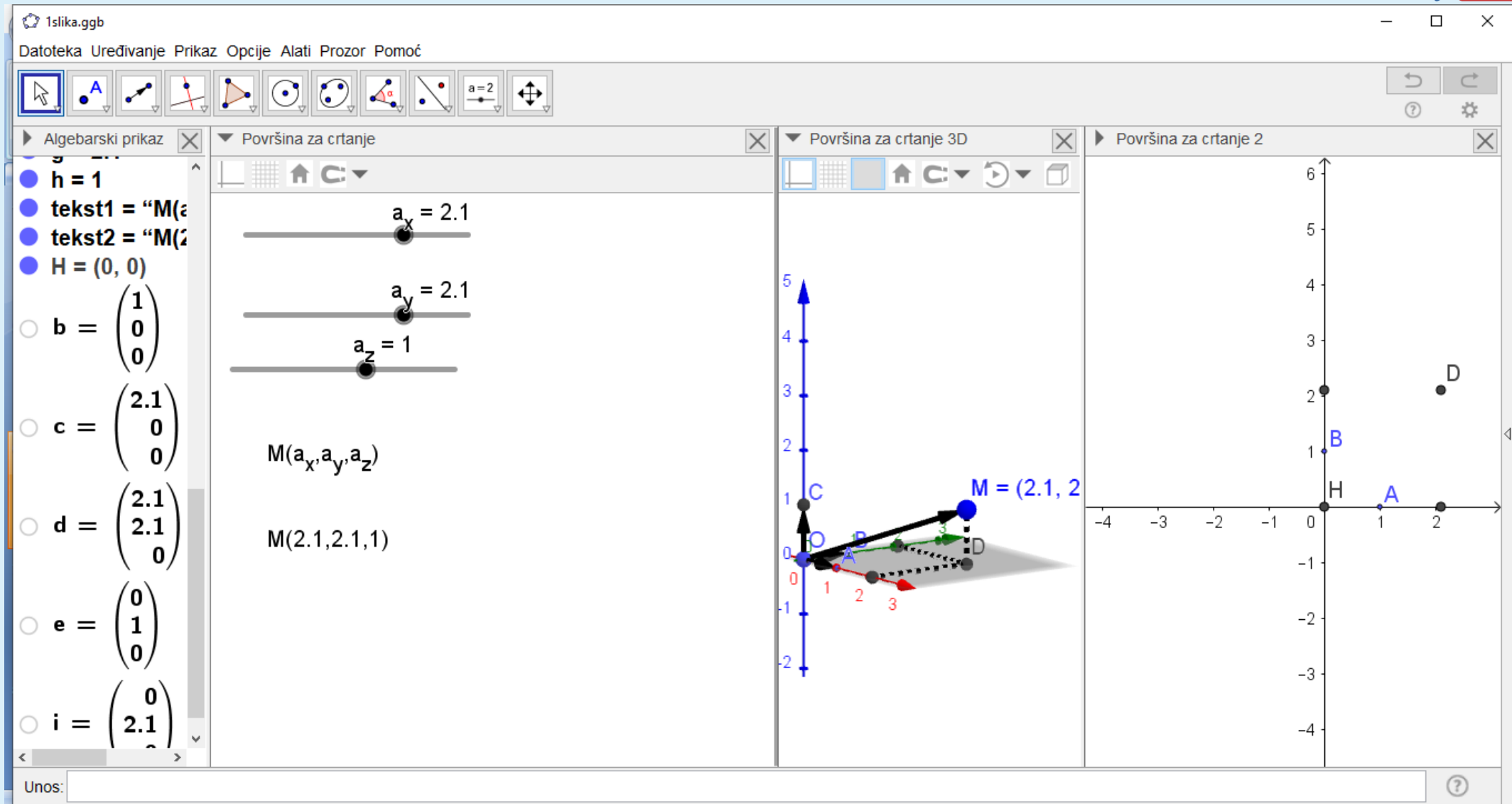
$a_y = 2.1$

$a_z = 1$

$M(a_x, a_y, a_z)$

$M(2.1, 2.1, 1)$

$M = (2.1, 2)$



The screenshot shows a dynamic geometry software window with four main panels. On the left is an 'Algebarski prikaz' (Algebraic view) panel containing a list of variables and vectors. The top center is a 'Površina za crtanje' (Drawing area) with three sliders for vector components a_x , a_y , and a_z . The bottom center is a 'Površina za crtanje 3D' (3D drawing area) showing a 3D coordinate system with a vector M and its projections onto the axes. The right panel is a 'Površina za crtanje 2' (2D drawing area) showing a 2D coordinate system with points A , B , C , D , and H plotted. The Windows taskbar is visible at the bottom.

VECTORS IN DINAMIC ENVIRONMENT

$$\vec{OM} = 3\vec{i} + (-2)\vec{j} + (1)\vec{k}, \quad \vec{ON} = 2.8\vec{i} + (1)\vec{j} + (2.2)\vec{k}, \quad \vec{OP} = -1\vec{i} + (1)\vec{j} + (4)\vec{k}$$

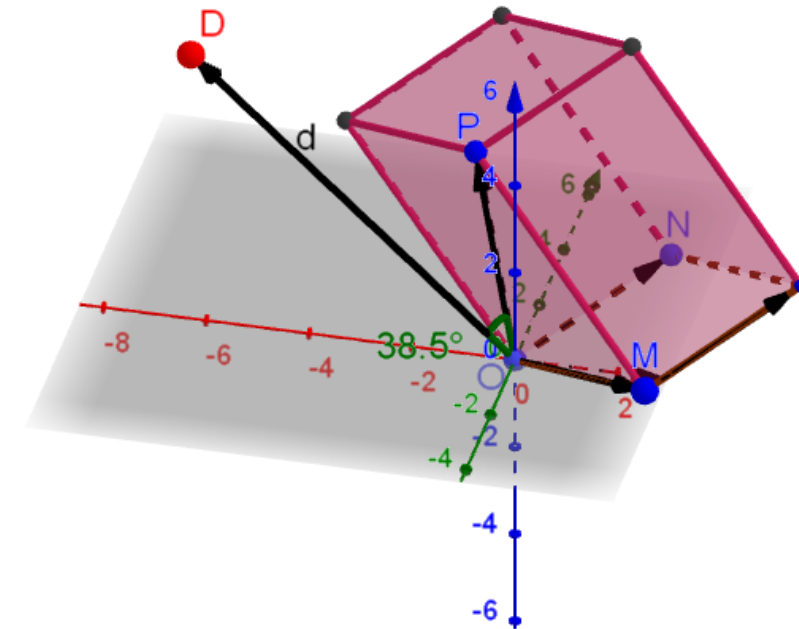
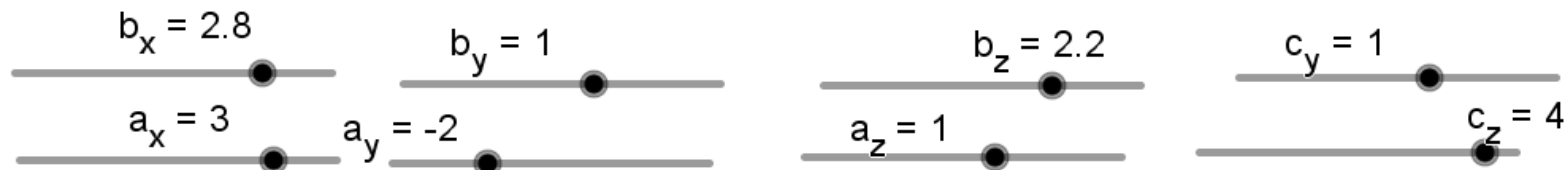
Mešoviti proizvod

$$(\vec{OM} \times \vec{ON}) \circ \vec{OP} = \vec{d} \circ \vec{OP} = |\vec{d}| |\vec{OP}| \cos(\angle(\vec{a}, \vec{d})) = P_p |\vec{OP}| \cos(\angle(\vec{OP}, \vec{d})) = 36$$

$$V = 36, \quad \alpha = 38.5^\circ$$

$$\vec{d} = \vec{OM} \times \vec{ON} = -5.4\vec{i} - (3.8)\vec{j} + (8.6)\vec{k}$$

$$(\vec{OM} \times \vec{ON}) \circ \vec{OP} = \begin{vmatrix} 3 & -2 & 1 \\ 2.8 & 1 & 2.2 \\ -1 & 1 & 4 \end{vmatrix} = 36$$



VECTORS IN DINAMIC ENVIRONMENT

Zapremina paralelopipeda -- Mešoviti proizvod vektora $\vec{a} \circ (\vec{b} \times \vec{c})$

Paralelopiped određen vektorima

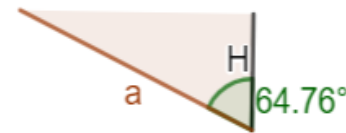
$$\vec{a} = \vec{OA} = \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} p \\ 3 \\ 1 \end{pmatrix}, \quad \vec{b} = \vec{OB} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \quad \vec{c} = \vec{OC} = \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ p \\ p \end{pmatrix}$$

$$P_O = 8.49 \quad V = 12$$

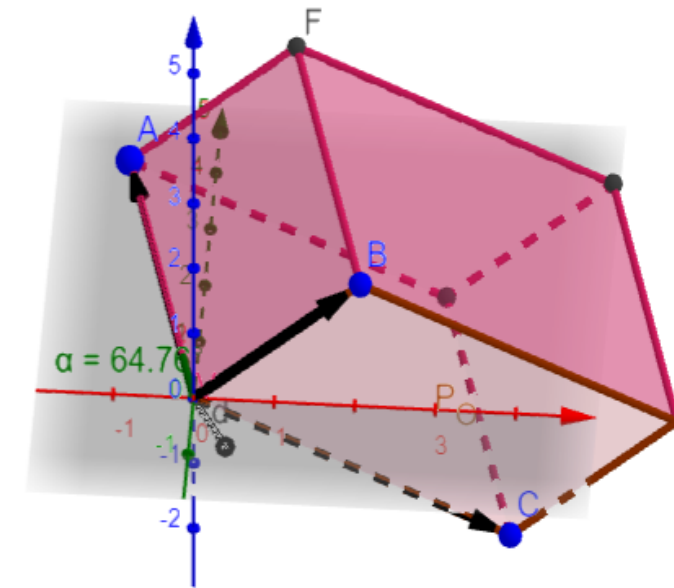
$$\vec{d} = (\vec{b} \times \vec{c}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 4 & -1 & -1 \end{vmatrix} = 6\vec{j} + (-6)\vec{k}$$

$$\vec{a} \circ (\vec{b} \times \vec{c}) = \begin{vmatrix} -1 & 3 & 1 \\ 2 & 1 & 1 \\ 4 & -1 & -1 \end{vmatrix} = 12$$

$$\begin{aligned} \vec{a} \circ (\vec{b} \times \vec{c}) &= \vec{a} \circ \vec{d} = |\vec{a}| |\vec{d}| \cos(\angle(\vec{a}, \vec{d})) = P_{\vec{b}, \vec{c}} |\vec{a}| \cos(\angle(\vec{a}, \vec{d})) = B * H \\ &= 8.49 * 3.32 * (0.43) = 12 \end{aligned}$$



$$\cos(\alpha) = \frac{H}{a}$$



p = -1

- 1
- 2
- 3
- 4V
- 5
- 6
- 7
- 8
- 9

Task: The vectors $\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$, $\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$ and $\vec{c} = c_x\vec{i} + c_y\vec{j} + c_z\vec{k}$ are given.

- Determine the vector $\vec{a} = \frac{\vec{a}}{2} + 2\vec{b} - \vec{c}$;
- Determine the surface area of the triangle forming the vectors \vec{a} and \vec{c} ;
- Determine the height corresponding to the side \vec{a} of the parallelogram formed by the vectors \vec{a} and \vec{b} ;
- Determine the volume of a parallelepiped formed by vectors \vec{a} , \vec{b} and \vec{c} ;
- Determine the equation of a plane that is parallel to the plane formed by the vectors \vec{a} and \vec{b} , and which contains a point $A(x_0, y_0, z_0)$.



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Površina pomoću integrala

Đurđica Takači

UNS Novi Sad

DEFINITE INTEGRAL IN DINAMIC ENVIRONMENT



$$f(x) = \frac{x^2}{4} + 1, \quad x \in [0, 4]$$

$$\Delta x = \frac{4}{n} = \frac{4}{4} = 1$$

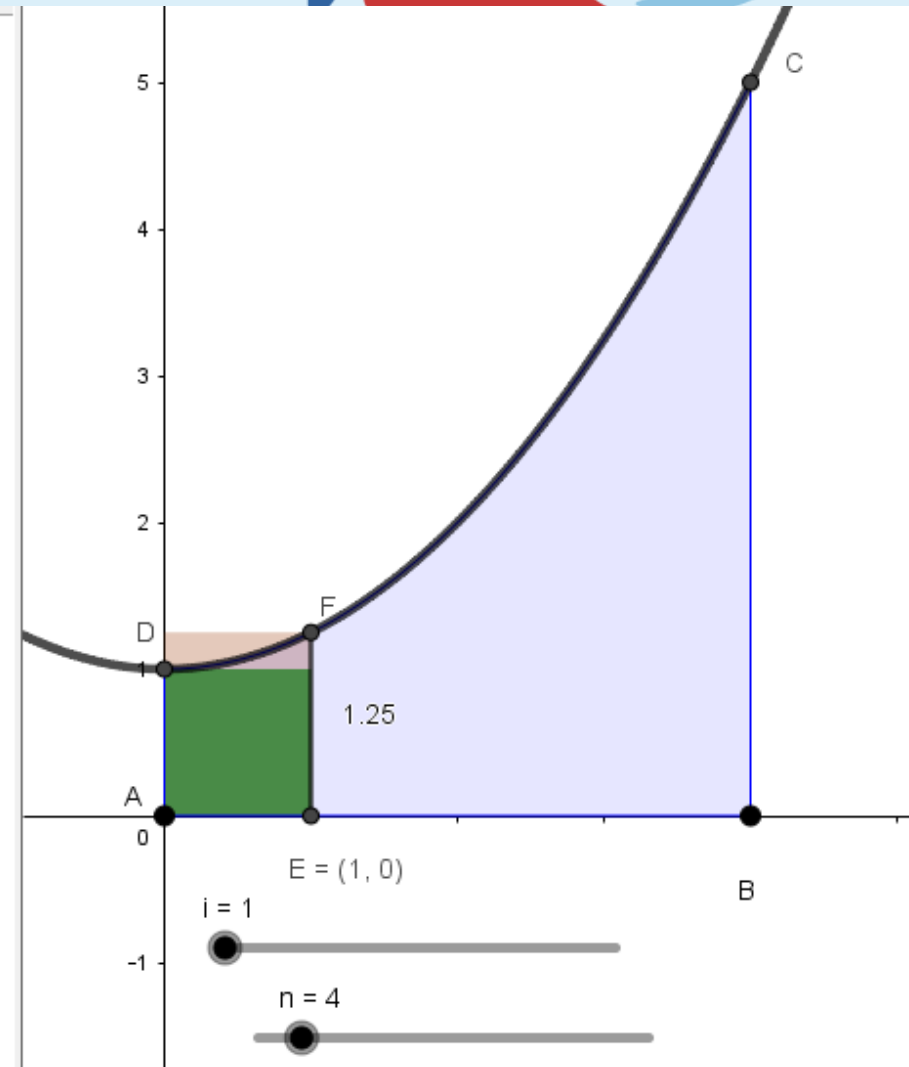
$$\sum_{i=1}^4 f(x_{i-1}) \Delta x = \Delta x(1 + 1.25 + 2 + 3.25) = 7.5$$

$$\sum_{i=1}^4 f(x_i) \Delta x = \Delta x(1.25 + 2 + 3.25 + 4) = 11.5$$

Šta se dešava ako se broj podintervala povećava?

Ako je $n=8$ odrediti zbir površina opisanih i upisanih pravougaonika.

- Up ZbirUp Op ZbirOp Pit



- $f(x) = \frac{2^x}{4} + 6$
- $a = -3$
- $b = 4$
- $g(x) = x(x+2) \frac{x-4}{10} + 2$
- $h(x) = \frac{2^x}{4} + 6 - \left(x(x+2) \frac{x-4}{10} + 2 \right)$
- $Of = 47.73$
- $Og = 9.51$
- $Ofg = 38.22$
- $F(x) = \frac{1}{2} x^2$
- $G(x) = \frac{1}{40} x^4 + \frac{-1}{15} x^3 + \frac{-2}{5} x^2 + 2x$
- $H(x) = \frac{1}{4} \cdot \frac{2^x}{\ln(2)} - \frac{1}{10} \left(\frac{1}{4} x^4 - \frac{2}{3} x^3 - 4x^2 \right)$
- $c_1 = 0$
- **tekst6** = " $F(x) = \int \left(\frac{2^x}{4} + 6 \right) dx = \frac{1}{2} x^2 + C$ "
- **tekst1** = " $\int_a^b f(x) dx = \int_{-3}^4 \left(\frac{2^x}{4} + 6 \right) dx = F$ "

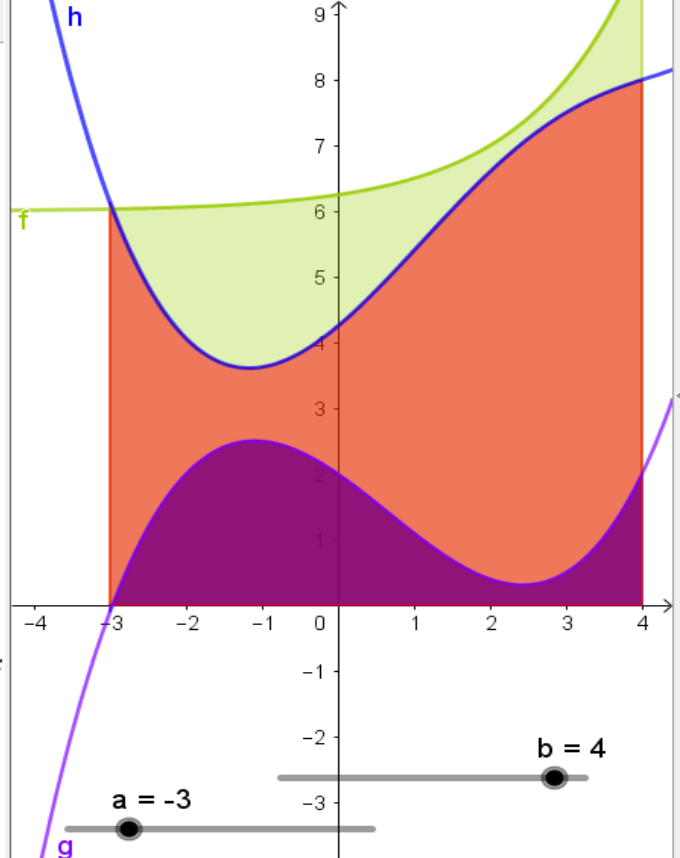
$$F(x) = \int \left(\frac{2^x}{4} + 6 \right) dx = \frac{1}{2} x^2 + C$$

$$\int_a^b f(x) dx = \int_{-3}^4 \left(\frac{2^x}{4} + 6 \right) dx = F(4) - F(-3) = 47.73$$

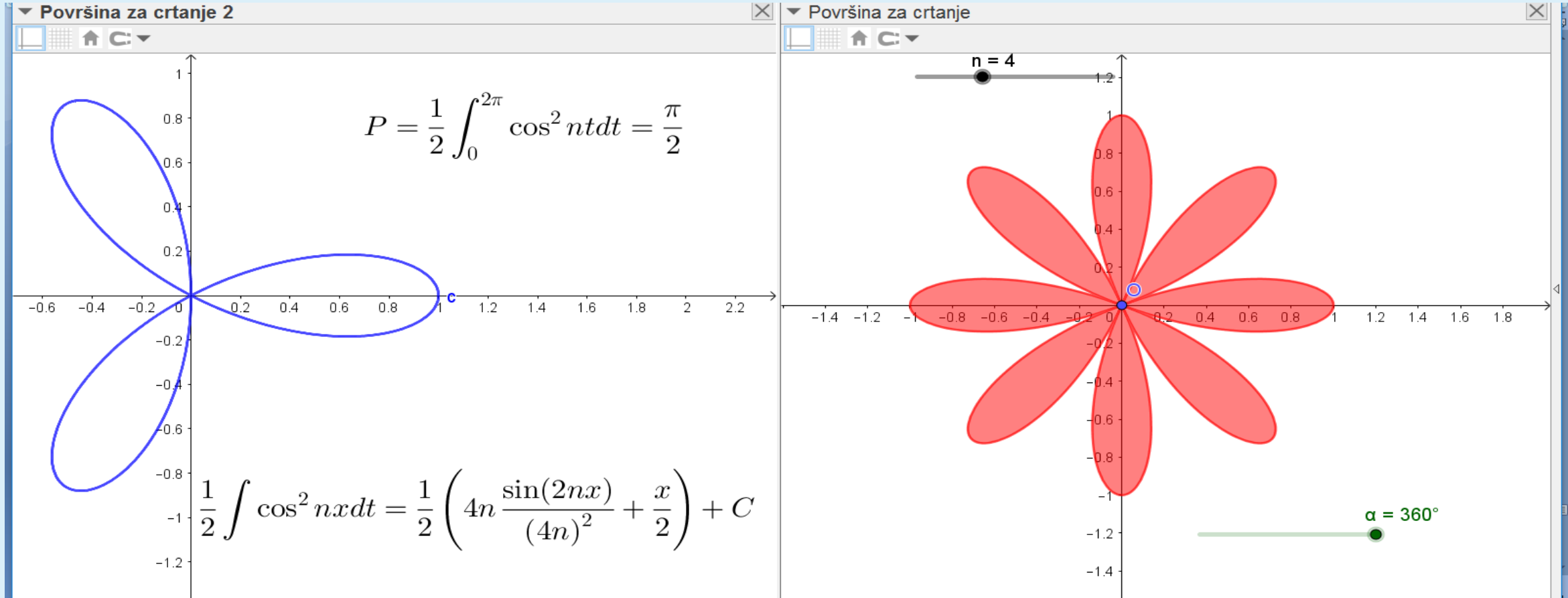
$$G(x) = \int g(x) dx = \frac{1}{40} x^4 + \frac{-1}{15} x^3 + \frac{-2}{5} x^2 + 2x + C,$$

$$\int_a^b g(x) dx = \int_{-3}^4 \left(x(x+2) \frac{x-4}{10} + 2 \right) dx = G(4) - G(-3) = 9.51$$

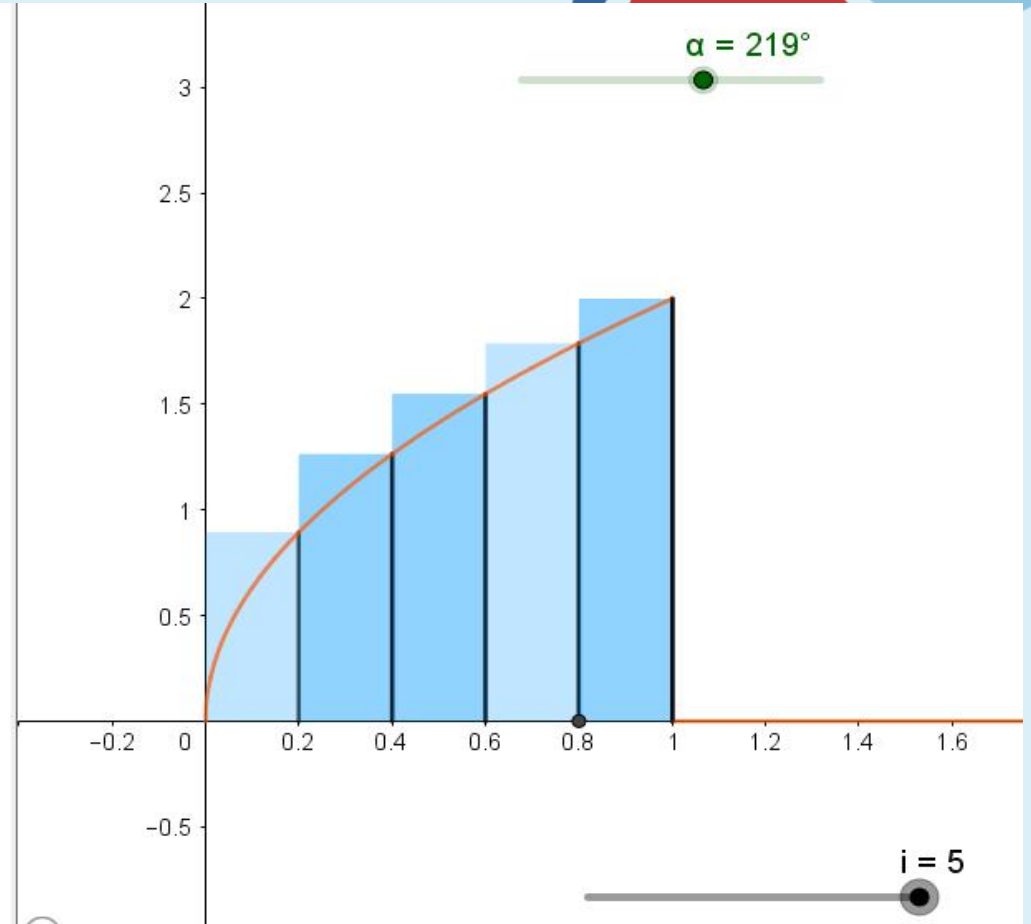
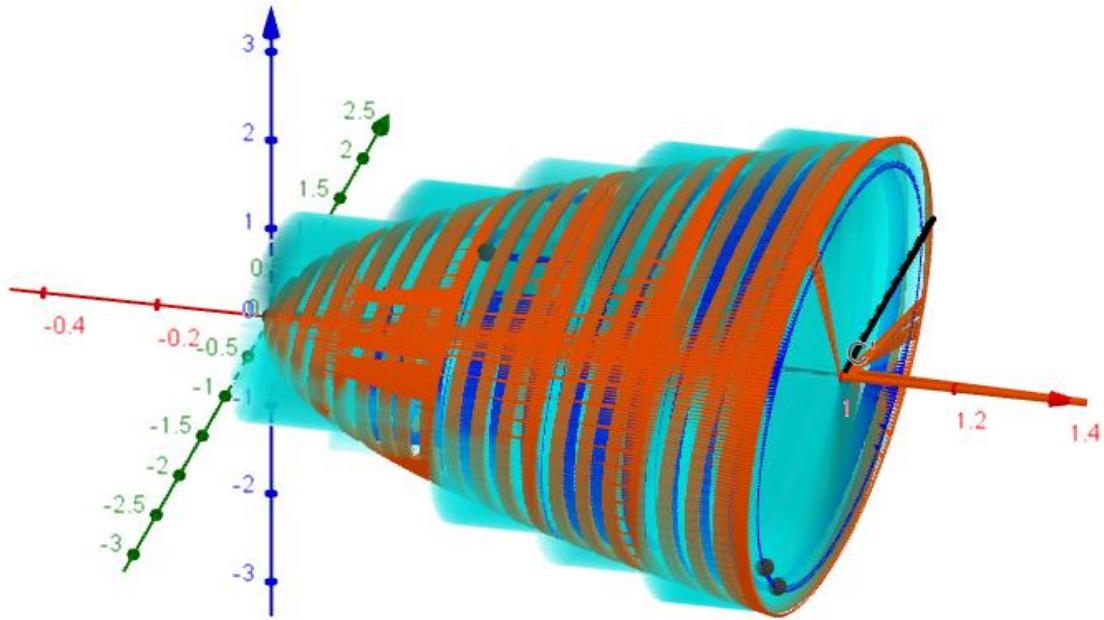
$$\int_a^b (f(x) - g(x)) dx = \int_{-3}^4 \left(\frac{2^x}{4} + 6 - \left(x(x+2) \frac{x-4}{10} + 2 \right) \right) dx = 38.22$$



The area

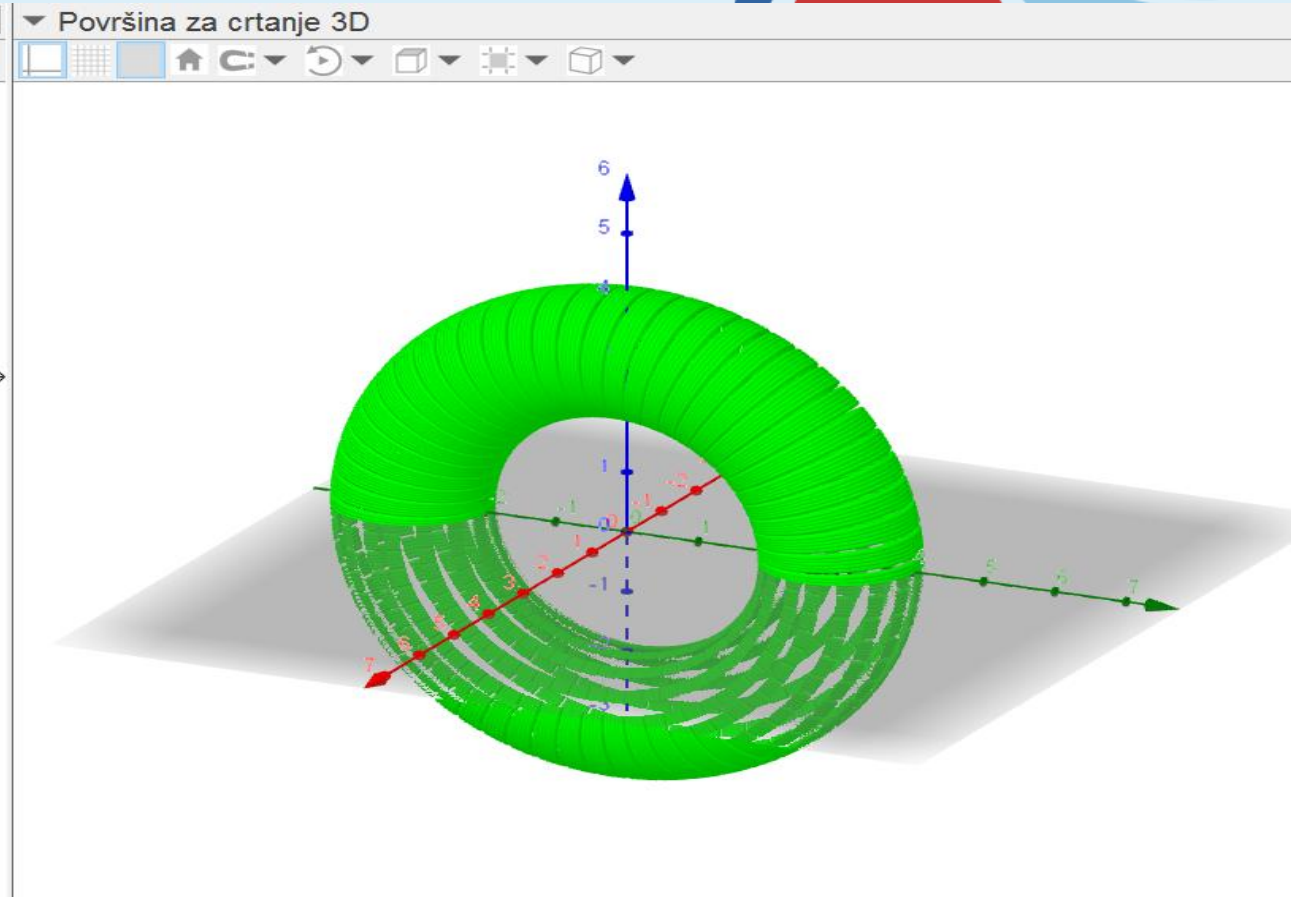
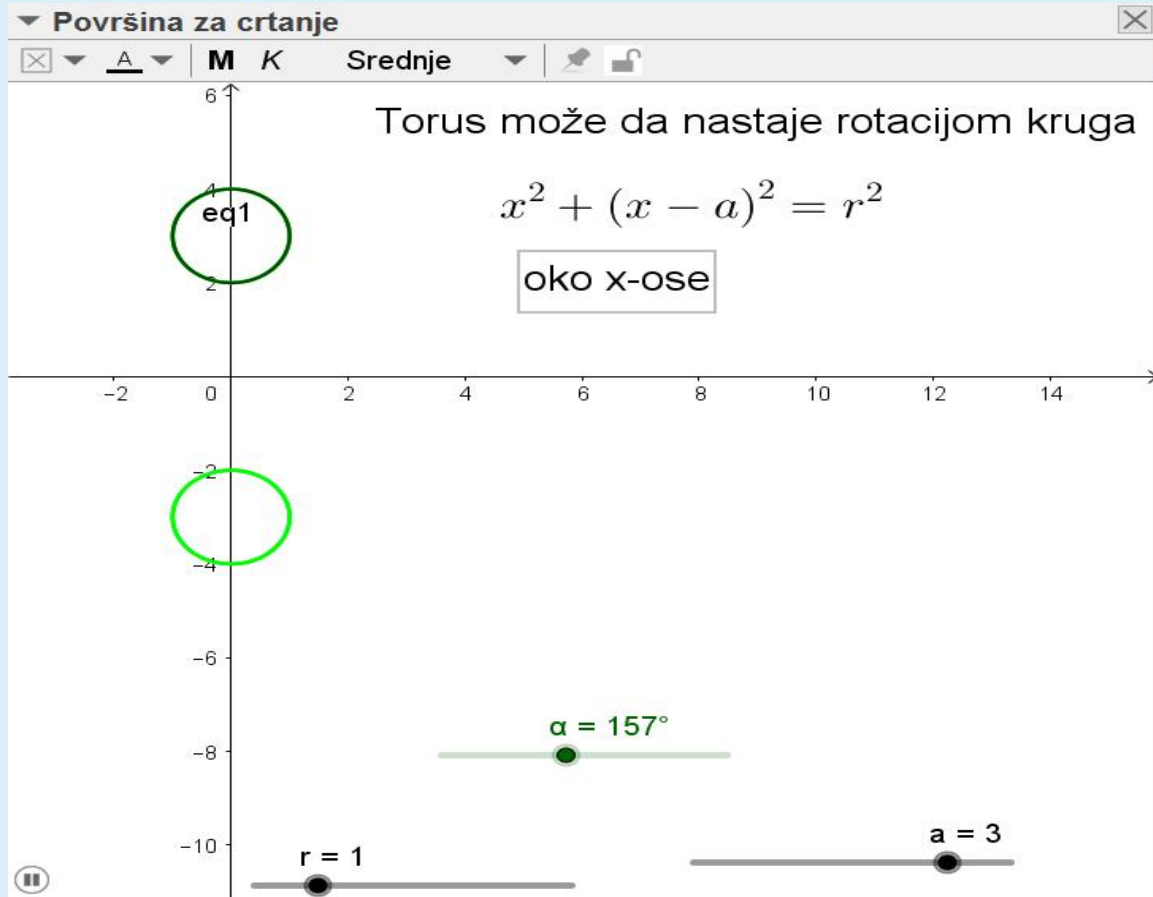


Volume of solid of revolution



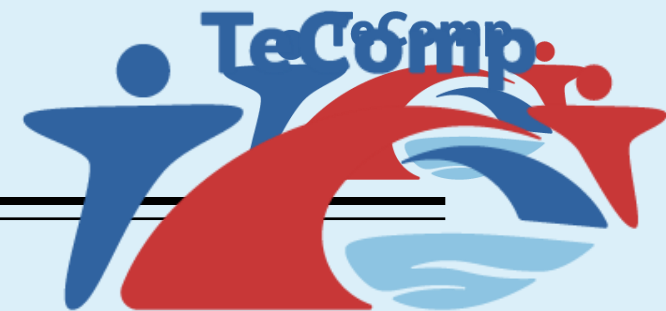
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