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The effects of application of dynamic software in calculus teaching and learning at university level

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Introduction

- Teachers' experiences support the fact that students often can have difficulties with understanding the concepts of calcuclus, especially multivariable calculus (such as multiple integrals).
- Need for new methodological approach(es) based on the dynamic software application for visualization the functions of one variable, multivariable functions, curves and surfaces in order to determine the area of integration of multiple integrals and boundaries for the variables, with the aim of successfully solving multiple integrals and their applications by students.

Introduction

Software packages used in classes

Year	2017/2018	2018/2019	2019/2020
Software packages	Wolfram Mathematica and GeoGebra	GeoGebra and GeoGebraTube	Graphing Calculator and 3D Calculator
Math content	Double integrals	Double and triple integrals	Double and triple integrals

Multiple representations

- The representation of the concept can be described as a “process of modeling concrete objects from the real, physical world into abstract ideas” (Hwang & Hu, 2013).
- By multiple representations we mean different ways of describing and symbolizing a certain term, as well as referring to that term.
- Representations are divided into:
  - external representations, which are located in the students environment and these are the representations that he perceives, and
  - internal representations, which are often called mental representations and by that term we mean the representations that the student creates in his mind.

Multiple representations

- Representations are divided into:
  - static representations, which include classical representations common in mathematics, such as formulas, tables, and figures, and
  - dynamic representations created under the influence of computer development and appropriate software packages.

Multiple representations

- Multiple representations and their connection reduce missunderstandings that are present when a mathematical concept is represented by only one representation (Goldenberg, 1988).
- Through the explicit, visual connections between different representations, students realize that by changing the properties of an object, represented by one type of representation, changes occur in other representations of the same concept, and cognitive connections between different representations becomes stronger (Kaput, 1986).

Constructivism

- The application of modern technology provides the teacher opportunity to organize quality teaching in a constructivist environment so that technology is used as a tool, ie. that its use does not become a learning goal (Taber, 2017), ie that technology in teaching should be used when its use is necessary and when it contributes to the quality of teaching and that in cases when the teacher estimates that its use would not improve teaching quality, it is not necessary to use.
- The implementation of technology in teaching, allows students to: manipulate content and data; to explore connections; to actively process the information; to construct knowledge in their mind so that they can share it with others (Jonassen, Peck & Wilson, 1999).

Visualization

- To visualize means to construct, create or establish connections between external (external) mathematical objects or their representations (diagrams, tables, images) and mental, ie internal images and to develop analytical thinking to improve the understanding of given mathematical concepts.
- Visualization is a technique that provides the opportunity for abstract mathematical concepts to be presented in a concrete way and for students to better understand abstract concepts through concrete representations (Macnab, Philips & Norris, 2012).
- Refusing visualization means giving up the roots of many of the essential mathematical ideas. In the early phase of the development of the theory of functions, limit values, continuity, derivatives, etc. visualization is the basic source of ideas (Tall, 1991).

Visualization

- The importance of visual thinking is so fundamental to understanding mathematical analysis that it is very difficult to imagine a successful course in mathematical analysis that does not adequately emphasize graphical representations of concepts and concepts that students become familiar with (Zimmermann, 1991).
- Candidates with a high level of spatial abilities in a large number of cases use graphical representation when solving tasks together with algebraic representation, and connecting two different ways of representation leads to student success in solving tasks (Sevimli & Delice, 2011).

Visualization

- CAS (Computer algebra system) is considered as a tool for visualization that provides establishing connections between different representations of mathematical concepts (Mallet, 2007).
- In studies related to the application of CAS in teaching and learning mathematical analysis, the focus is mainly on:
  - how to apply software packages in math classes;
  - the influence of CAS on students' attitudes, interests and motivation;
  - development of operational and conceptual knowledge by students.

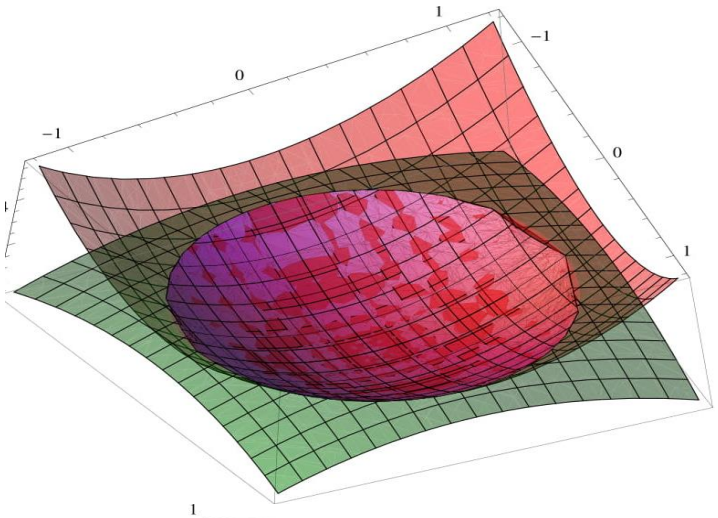
Visualization

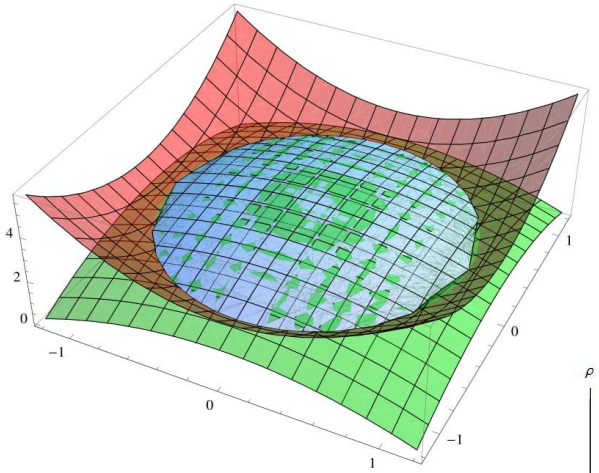
- A greater degree of interaction for visualizing concepts improves understanding and leads to students' deeper and better understanding.
- Teaching supported by the use of CAS can significantly help connecting the symbolic with the visual representation of mathematical concepts and concepts so that students can use visual or analytical thinking (reasoning) when solving tasks and problems (Sevimli, 2013).
- Students who were exposed to computer-assisted instruction gained a lot of self-confidence in math classes and showed a lower degree of anxiety toward math, as opposed to students who adopted the given concepts in the traditional way (Glickman, 2000).

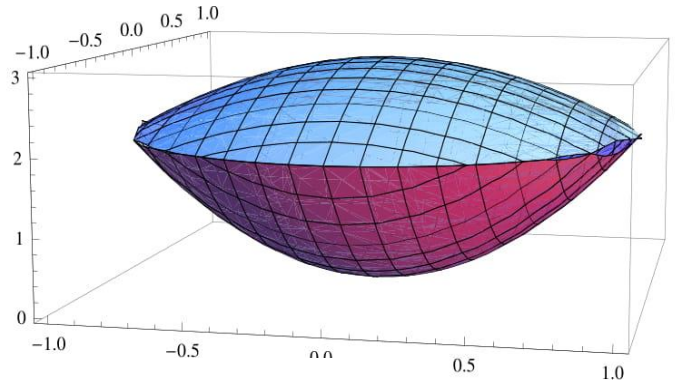
The first approach

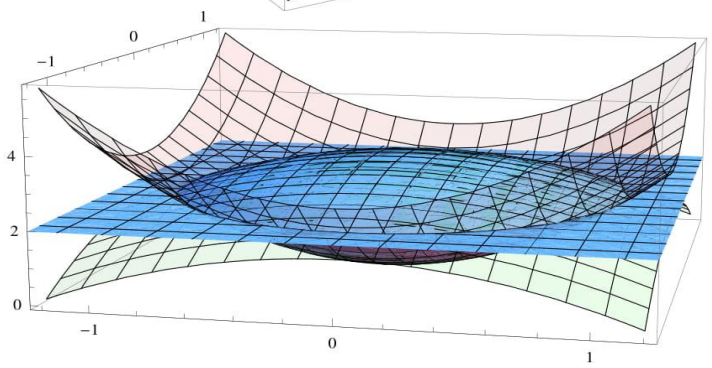
- Dynamic materials, ie graphs of multivariable functions, as well as graphs of surfaces in space given by appropriate equations were created in the Wolfram Mathematica software package.
- Students had the opportunity to individually move and rotate images of surfaces and their intersections, to change the parameter's value and to observe the influence of that change in the object properties on the corresponding graph.

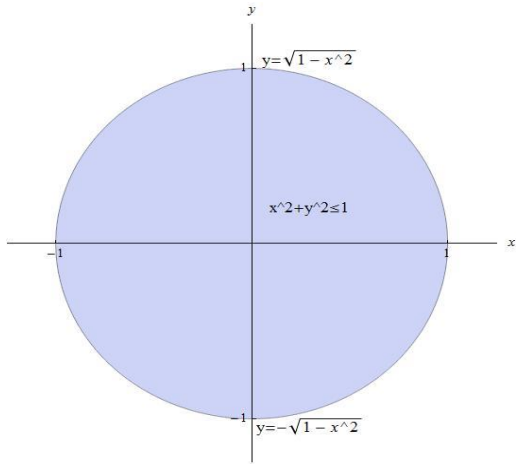
The first approach

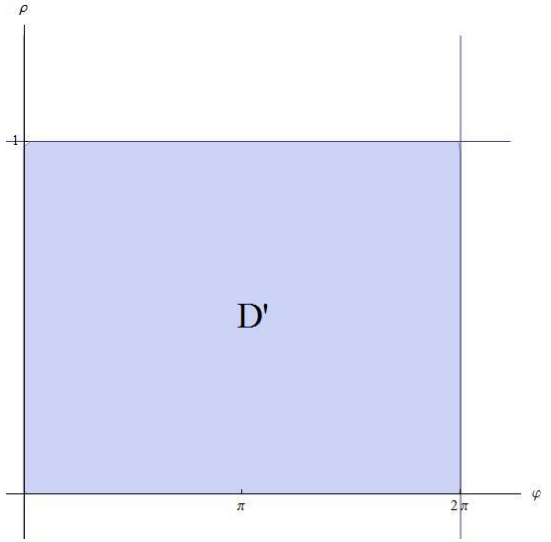












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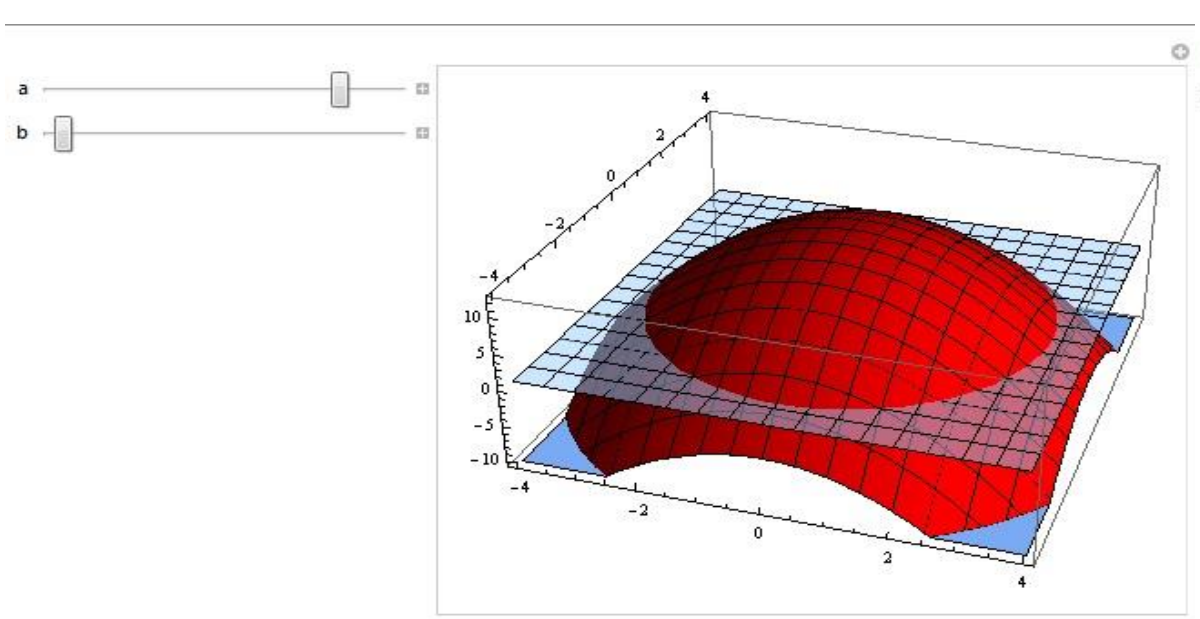
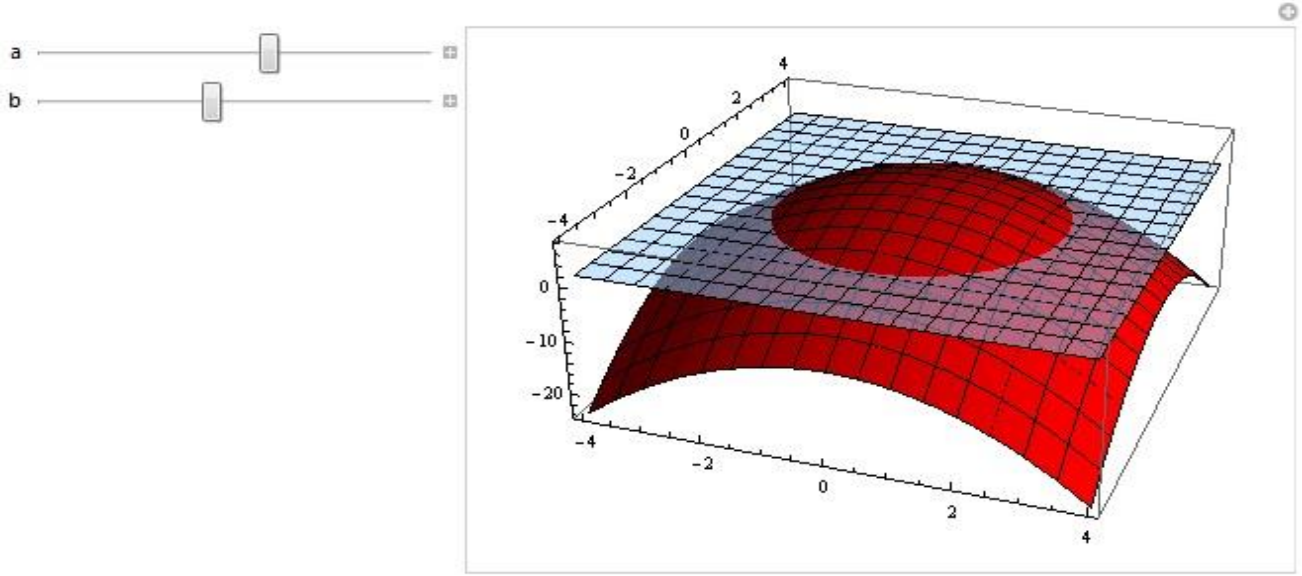




The second approach

- Dynamic materials, ie graphs of functions (single variable or multivariable functions), as well as graphs of surfaces given by appropriate equations were created in the GeoGebra, and then uploaded on the GeoGebraTube platform.

The second approach

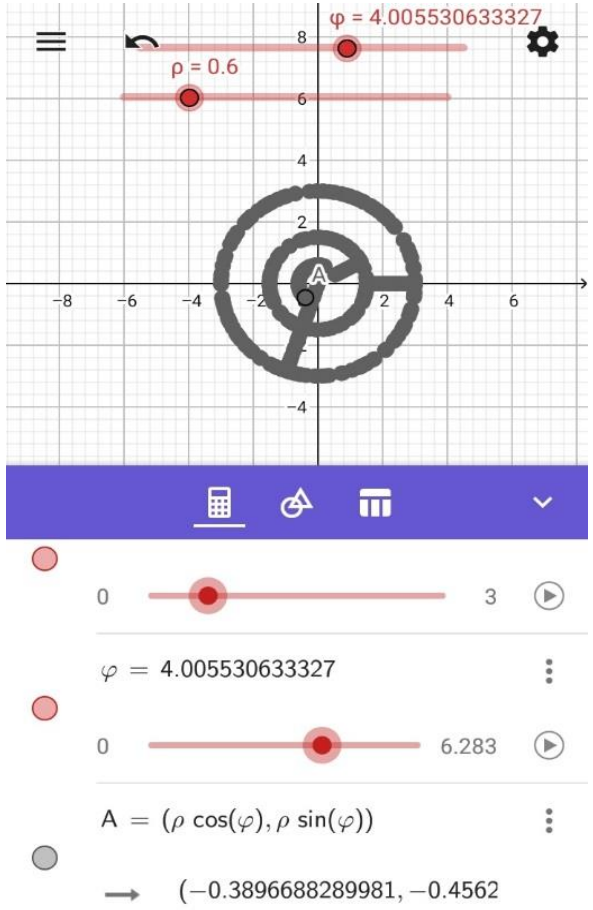
Students' attitudes towards this teaching and learning approach

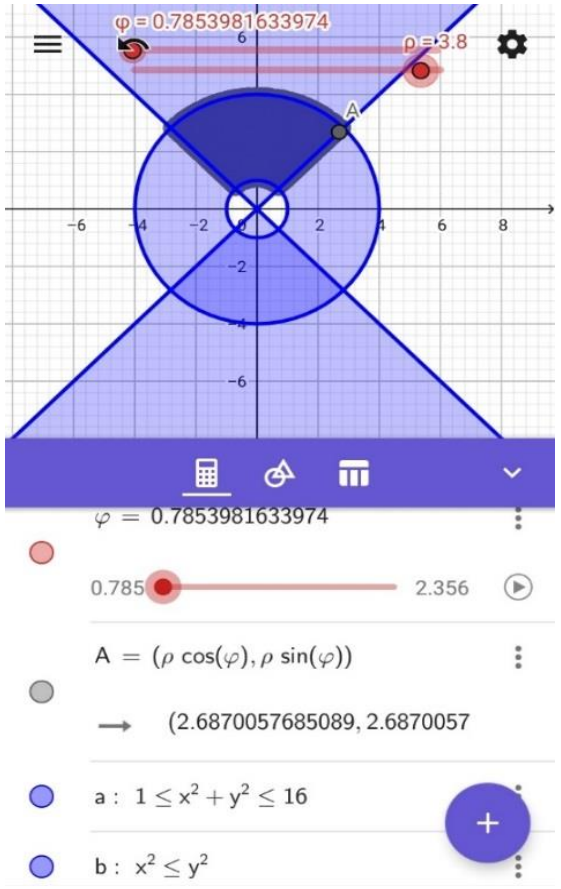
- The highest degree of agreement with the statements is present in the statements:
  - I liked that with the help of a mobile device I was able to observe the graphics of functiones, lines, planes and surfaces and their intersectiones;
  - I think that with the help of a mobile device, I would more easily and quickly adopt the math concepts related to the definite integral;
  - I think that with the help of a mobile device, I would be able to adopt concepts related to analytic geometry (lines, planes, spheres and their mutual relations) in easier and faster way;
  - I think that with the help of a mobile device, it would be easier and faster to adopt content related to the integration of functions of more variables (line and surface integral).

The third approach

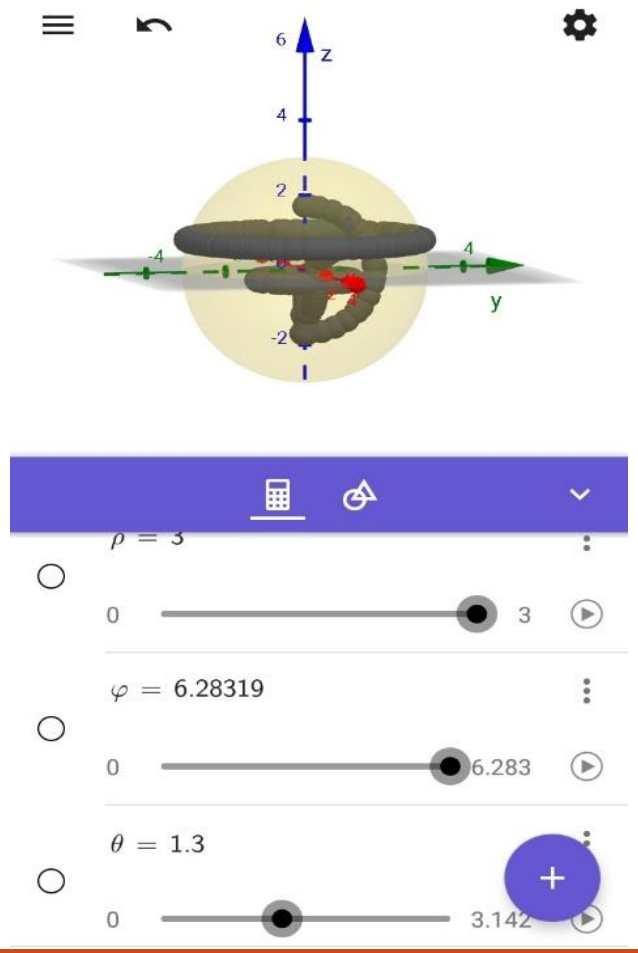
- Students are acquainted with the dual nature of applications *Graphing calculator* and *3D Calculator*, that by writing equations (or inequalities) of mathematical objects (algebraic representation), graphical representations of given mathematical objects are obtained.
- Students created appropriate teaching materials for developing their theoretical knowledge and while solving specific problems from multiple integrals.

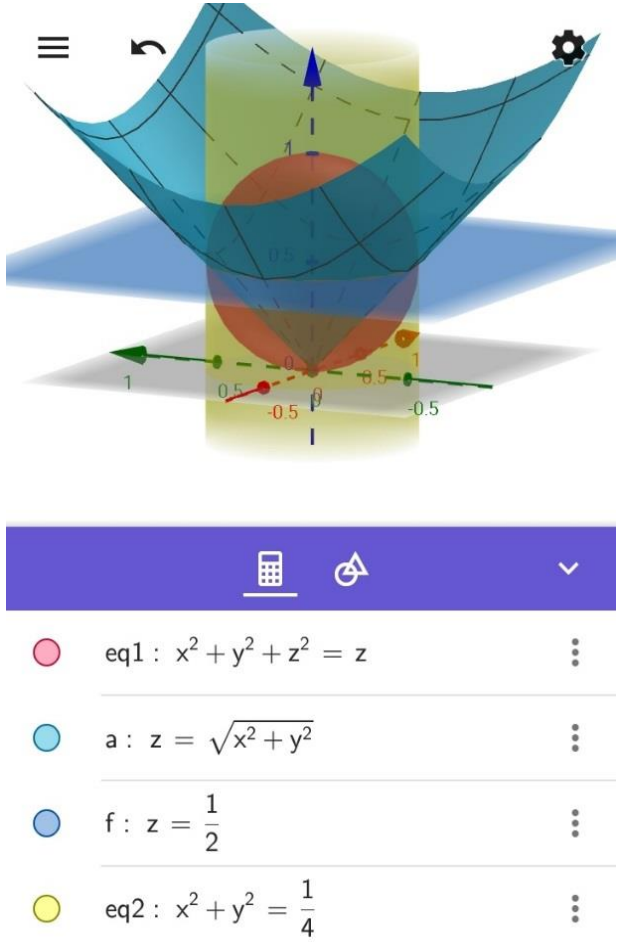
The third approach





The third approach





Conclusion

- The usage of the described methodological approaches based on the integration of the application of dynamic software for the visualization of multiple integrals contributes to a better quality of students' theoretical, visual and procedural knowledge and skills related to multiple integrals.
- The application of dynamic software *Wolfram Mathematica, GeoGebra, Graphing Calculator* and *3D Calculator* enables better visualization of mathematical concepts (in plane and in space) and dynamic connection of algebraic and graphical representations of those objects.
- The application of a constructivist form of teaching and learning in terms of providing students with the opportunity to independently create digital teaching materials and actively adopt the teaching contents also leads to a better knowledge and understanding of these contents by students.

Thank you for your attention!