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Leonardo Da Vinci, credit cards, snails ...

flower, galaxies, faces ...

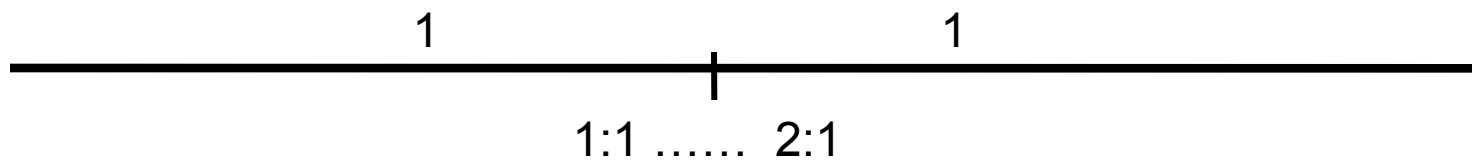
1,618 033 988 749 894 840 20....

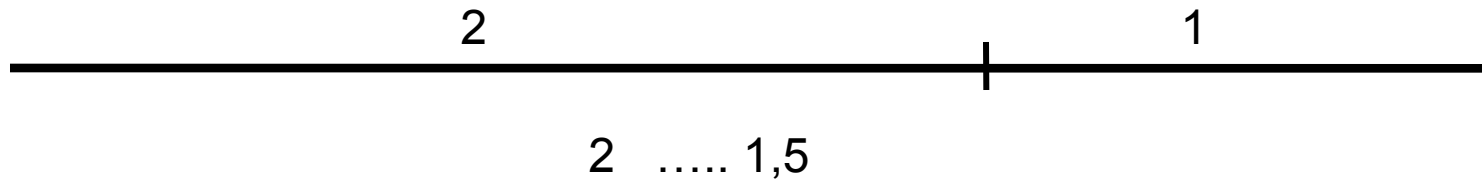
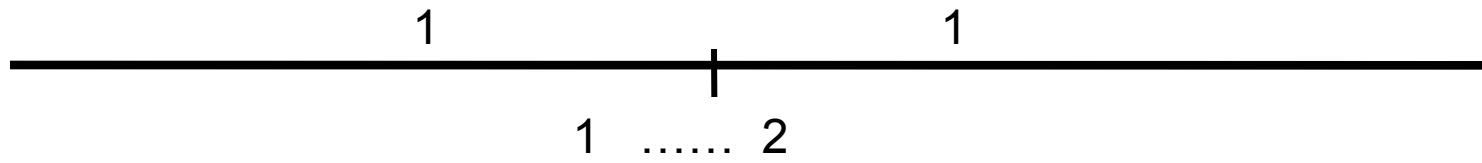


Best known constants

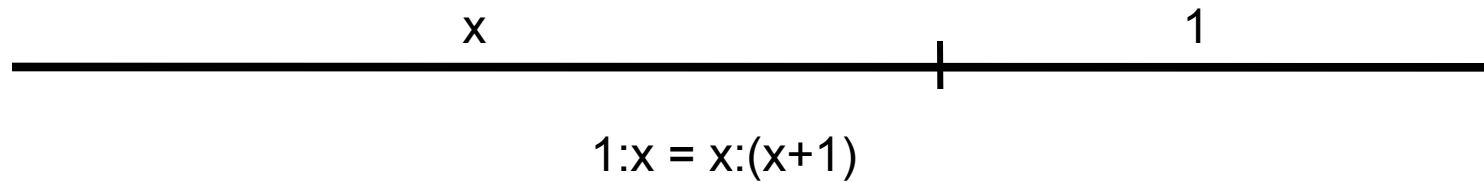
- $\pi = 3,141\ 59\dots$
- $e = 2,718\ 28\dots$
- $i = \sqrt{-1}$
- 0
- 1

- $e^{i\pi} + 1 = 0$





Golden section



$x = 1,618\ 033\ 988\ 749\ 894\ 840\ 20\dots$

$$= \frac{1 + \sqrt{5}}{2}$$

$$= \varphi$$

Try with your calculator

- φ^2
- $1 / \varphi$

Try with your calculator

- $\varphi^2 = 1 + \varphi$

- $1 / \varphi$

Try with your calculator

- $\varphi^2 = 1 + \varphi$
- $1 / \varphi = \varphi - 1$

Golden section as a chain fraction

$$\varphi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}}$$

$$\varphi = 1 + \frac{1}{\varphi}, \frac{1}{\varphi} = \varphi - 1$$

Golden section as a root

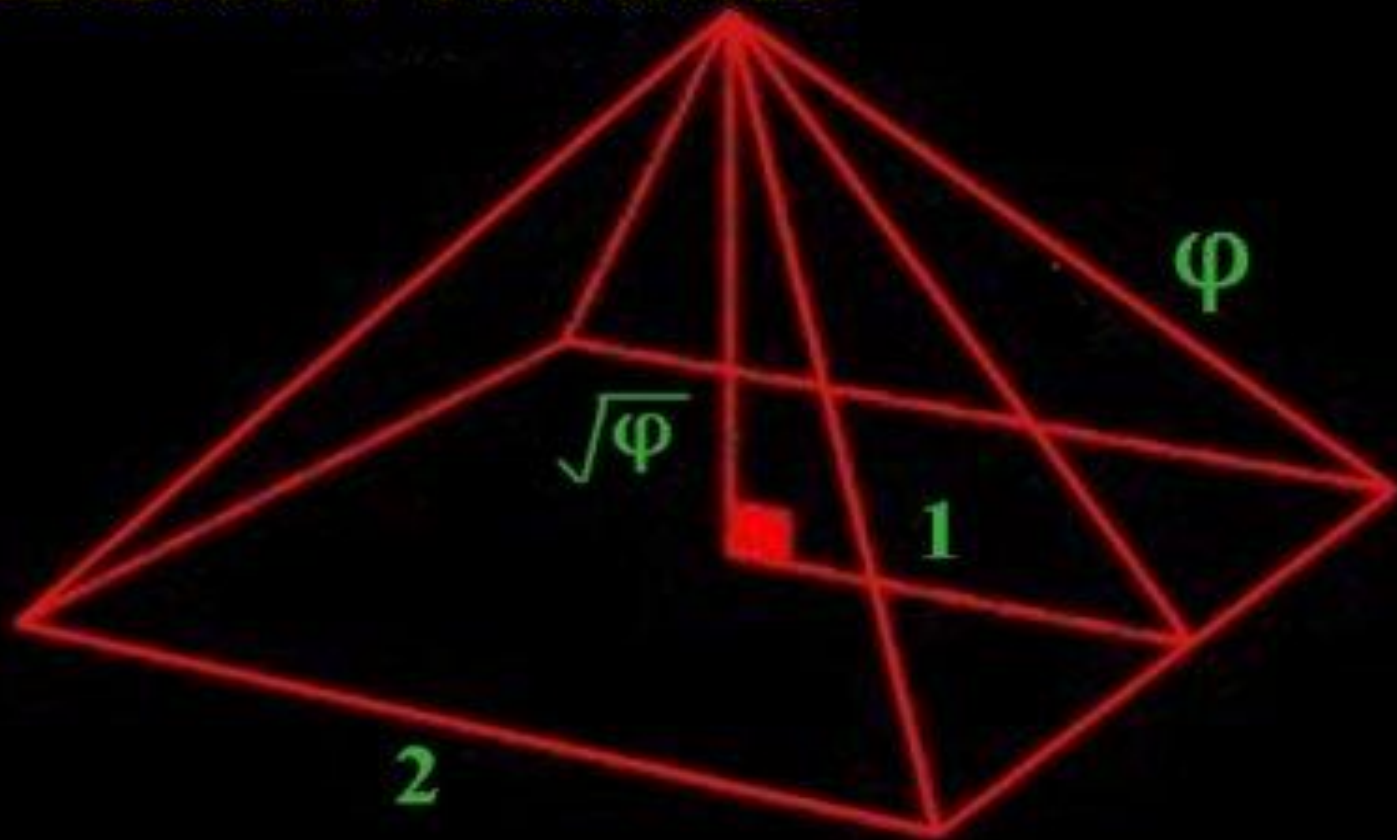
$$\varphi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}}$$

$$\varphi^2 = 1 + \varphi$$

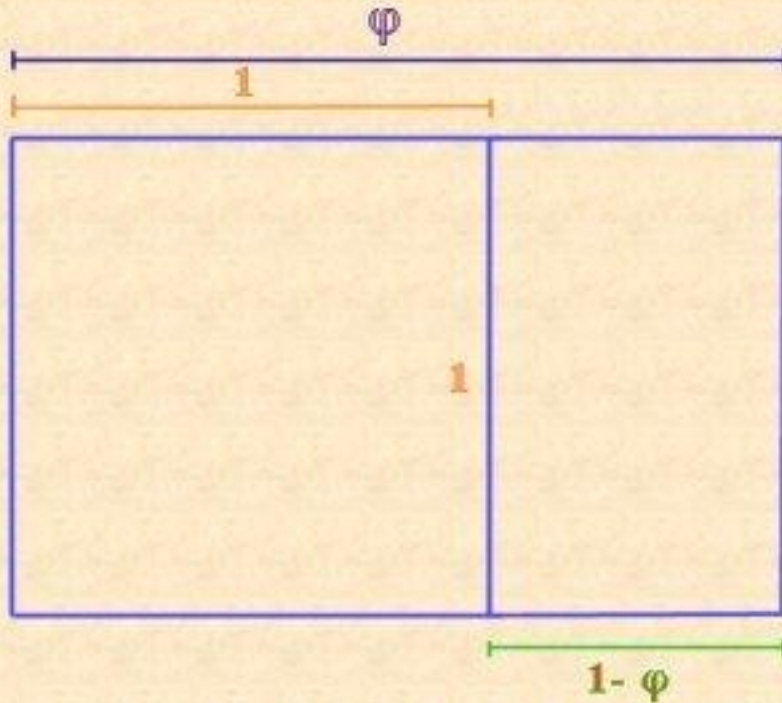
Leonardo Da Vinci



GREAT PYRAMID



The golden rectangle



$$\frac{\varphi}{1} = \frac{1}{\varphi - 1}$$

$$\varphi(\varphi - 1) = 1$$

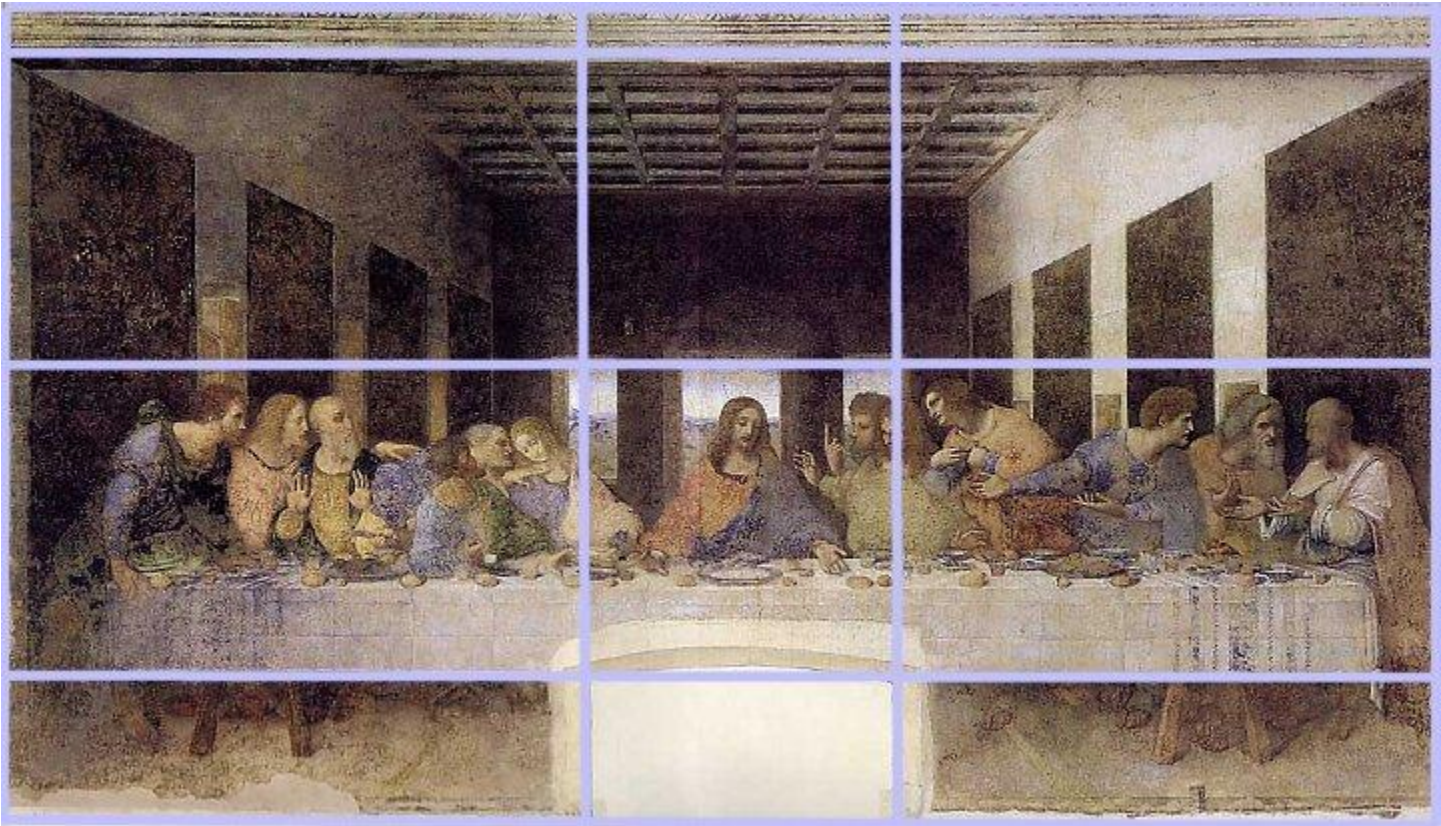
$$\varphi^2 - \varphi - 1 = 0$$

$$\varphi = \frac{1 + \sqrt{5}}{2}$$

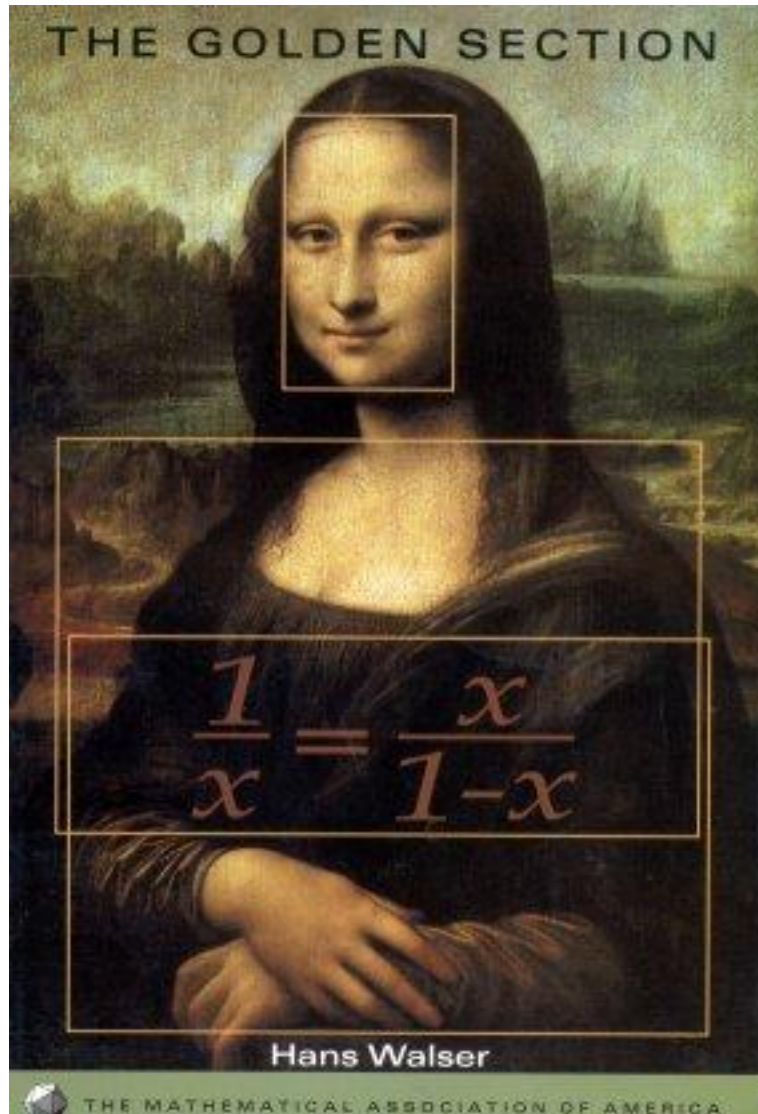
$$\varphi = 1.618033989\dots$$

(not commensurable)

The Golden Rectangle



THE GOLDEN SECTION



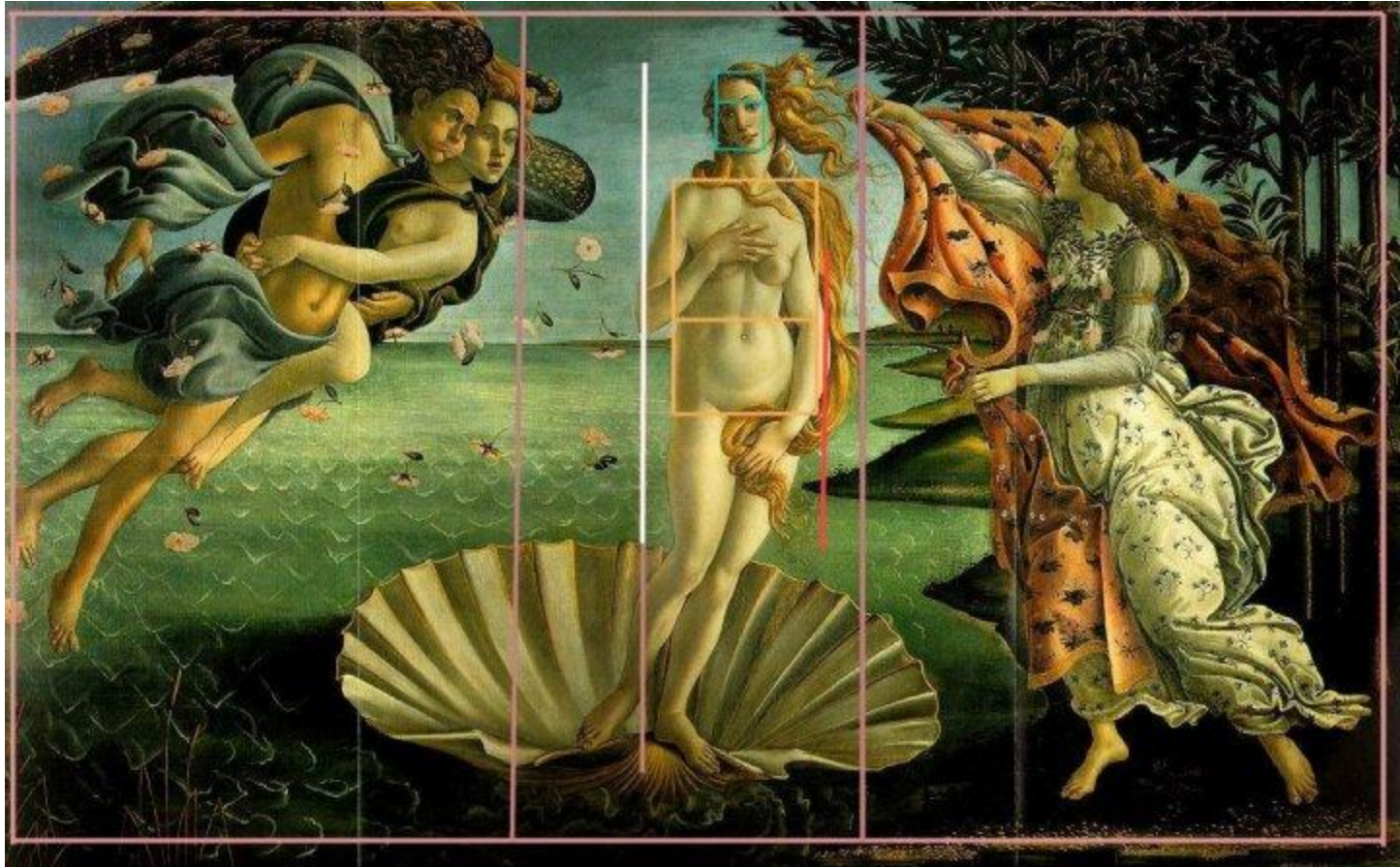
$$\frac{1}{x} = \frac{x}{1-x}$$

Hans Walser



THE MATHEMATICAL ASSOCIATION OF AMERICA

Sandro Boticelli



mBank 



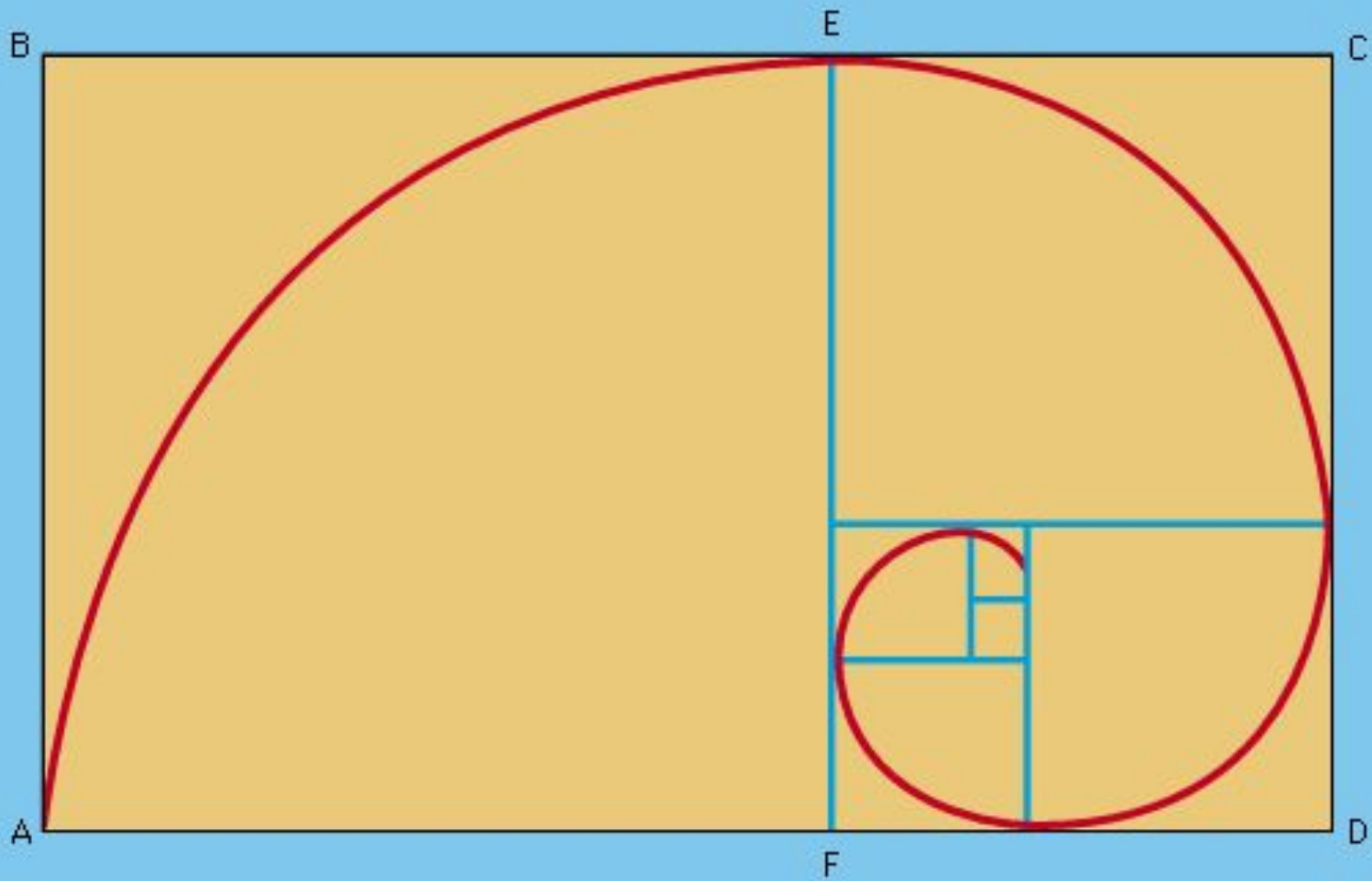
4000 1234 5678 9010

4461

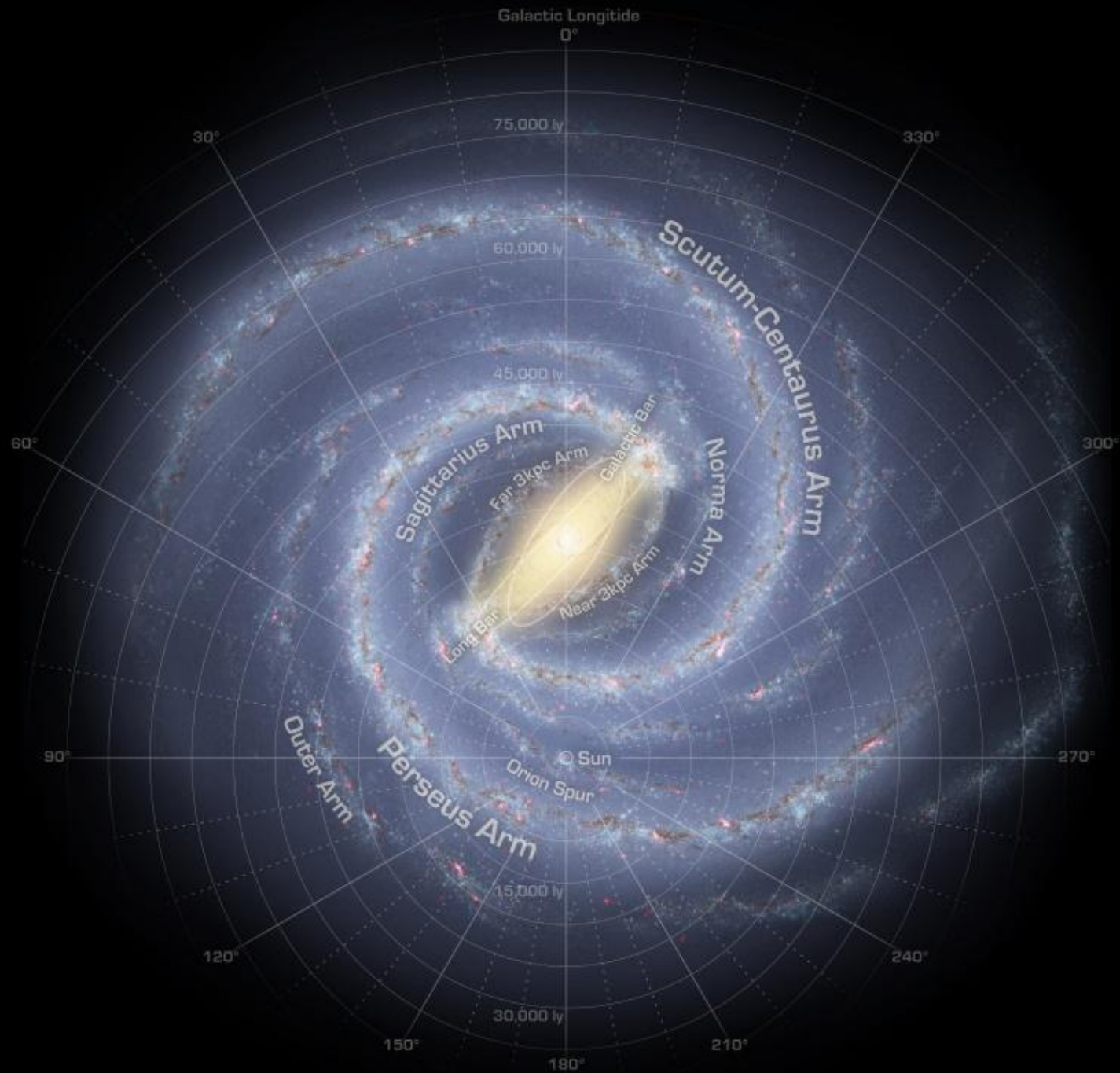
EXPIRES MONTH YEAR
END ▶

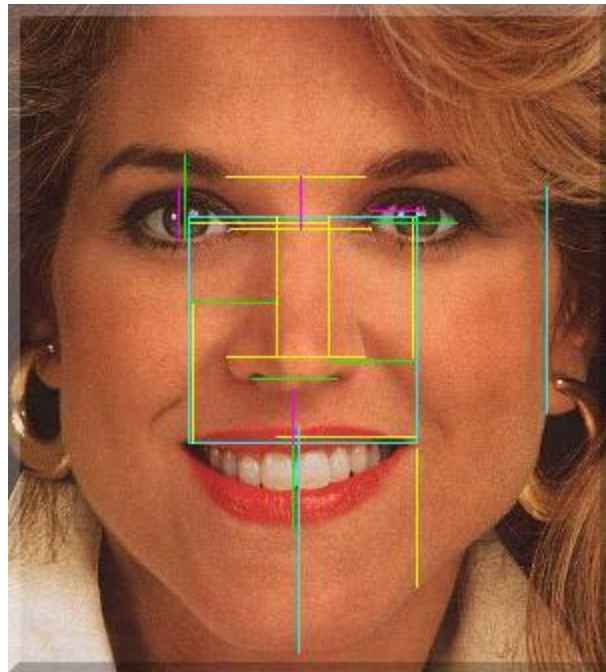
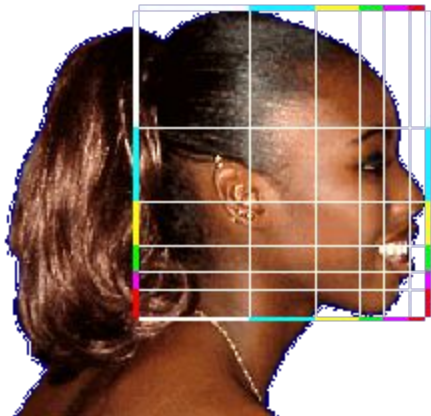
MAXIMILÁN VÝHODNÝ





Milky way







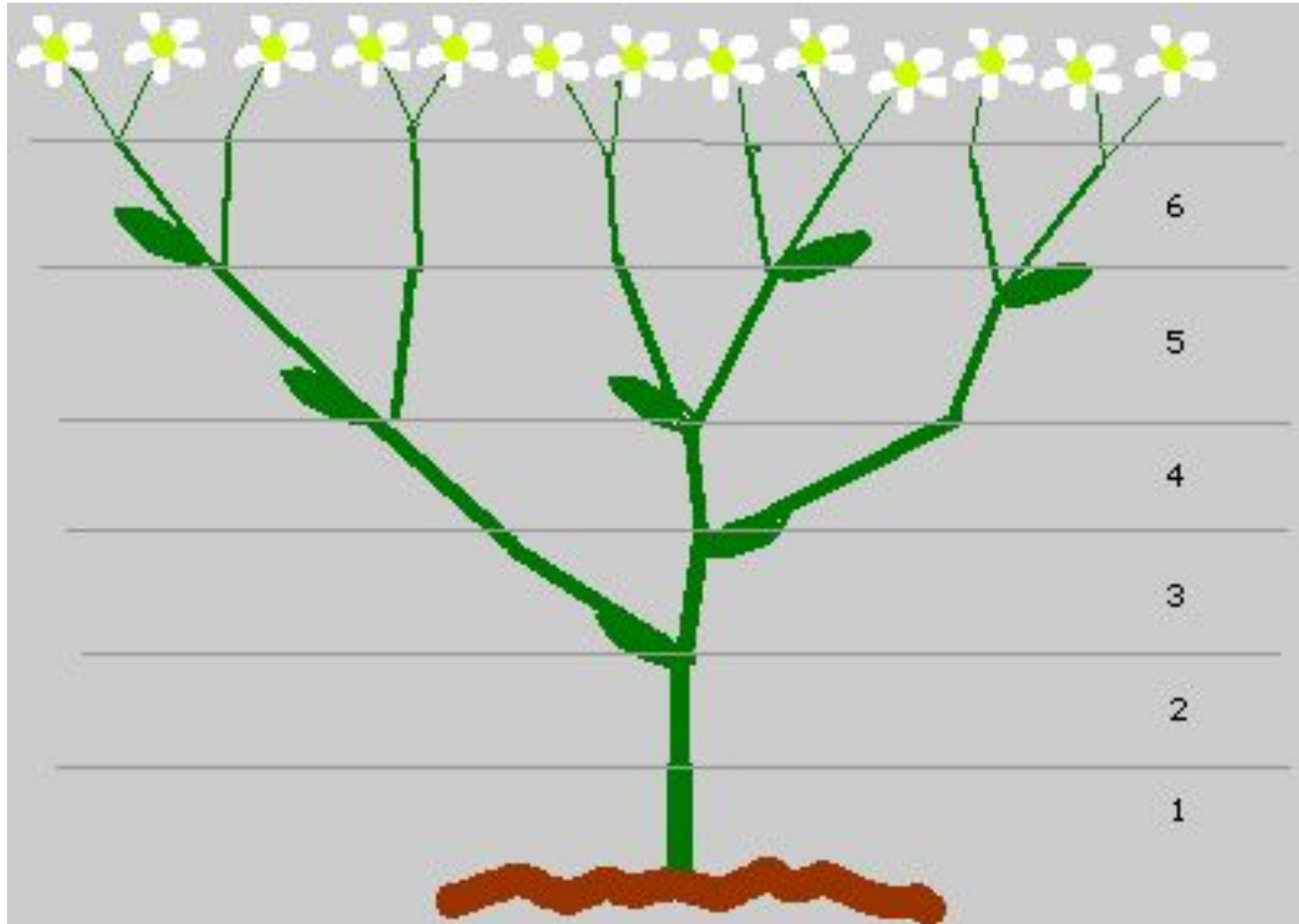
Saturn



Fibonacci numbers

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233,
377, 610, 987, 1597, 2584, 4181, 6765,
10946, 17711, 28657, 46368, 75025,
121393, 196418, 317811, 514229,
832040, 1346269, 2178309, 3524578,
5702887, 9227465, 14930352, 24157817,
39088169,

Fibonacci flowers



Numbers of flower leaves

- lily, iris: 3
- rose, strawberry: 5
- driada (de ocho pétalos): 8
- la margareta: 21
- králik: 34





last decimals:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...

0, 1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0, 7, ...

-repeat with cycle 60

last two digits:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...

- cycle **300**

last 3 digits:

- cycle **1,500**

last 4 digits:

- cycle **15,000**

last 5 digits:

cycle **150,000**

etc.

Fibonacci numbers and golden section

- 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169
- **find F_{k+1} / F_k**

