Fractals

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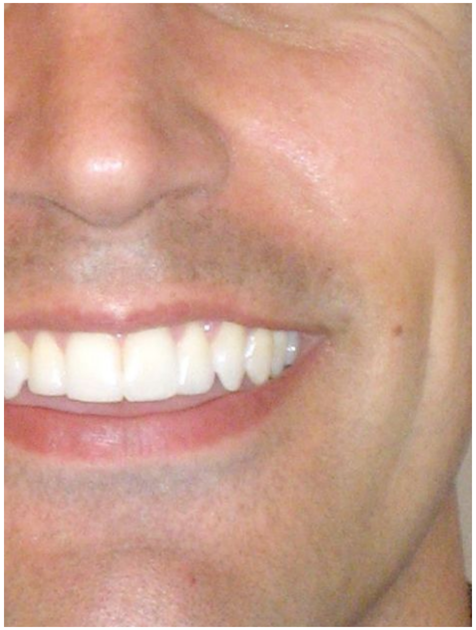
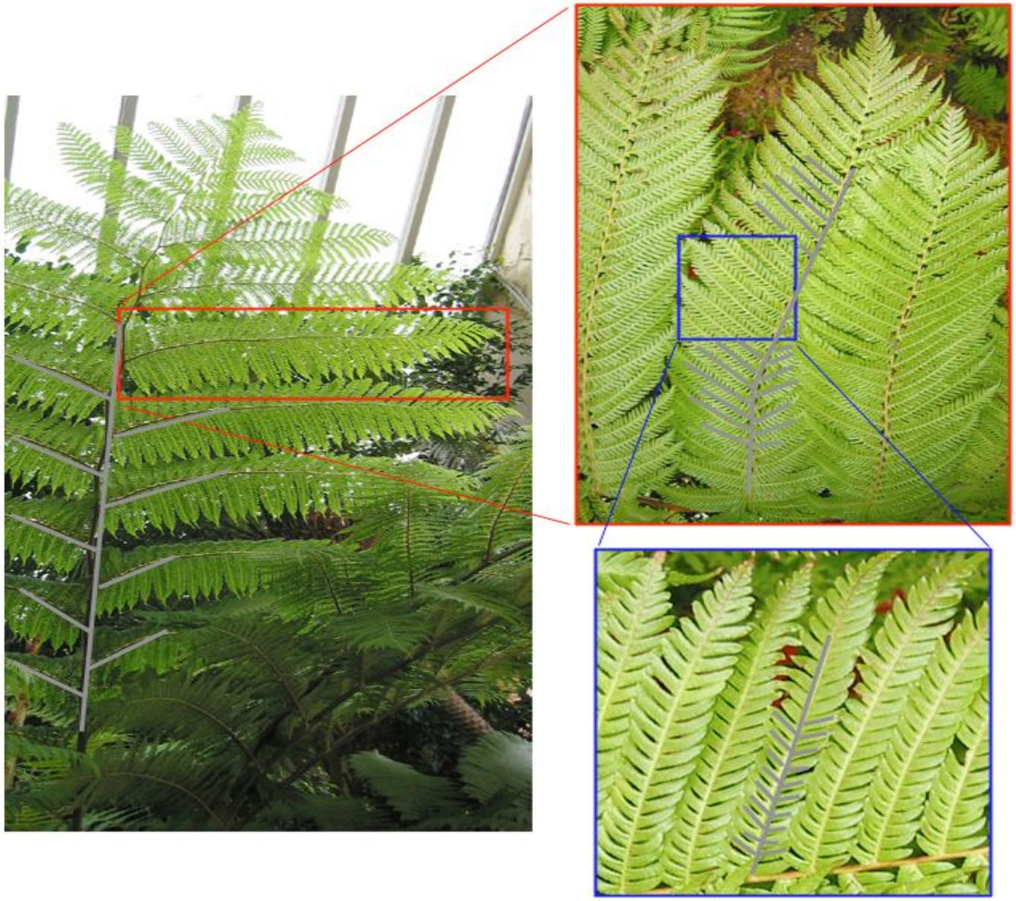
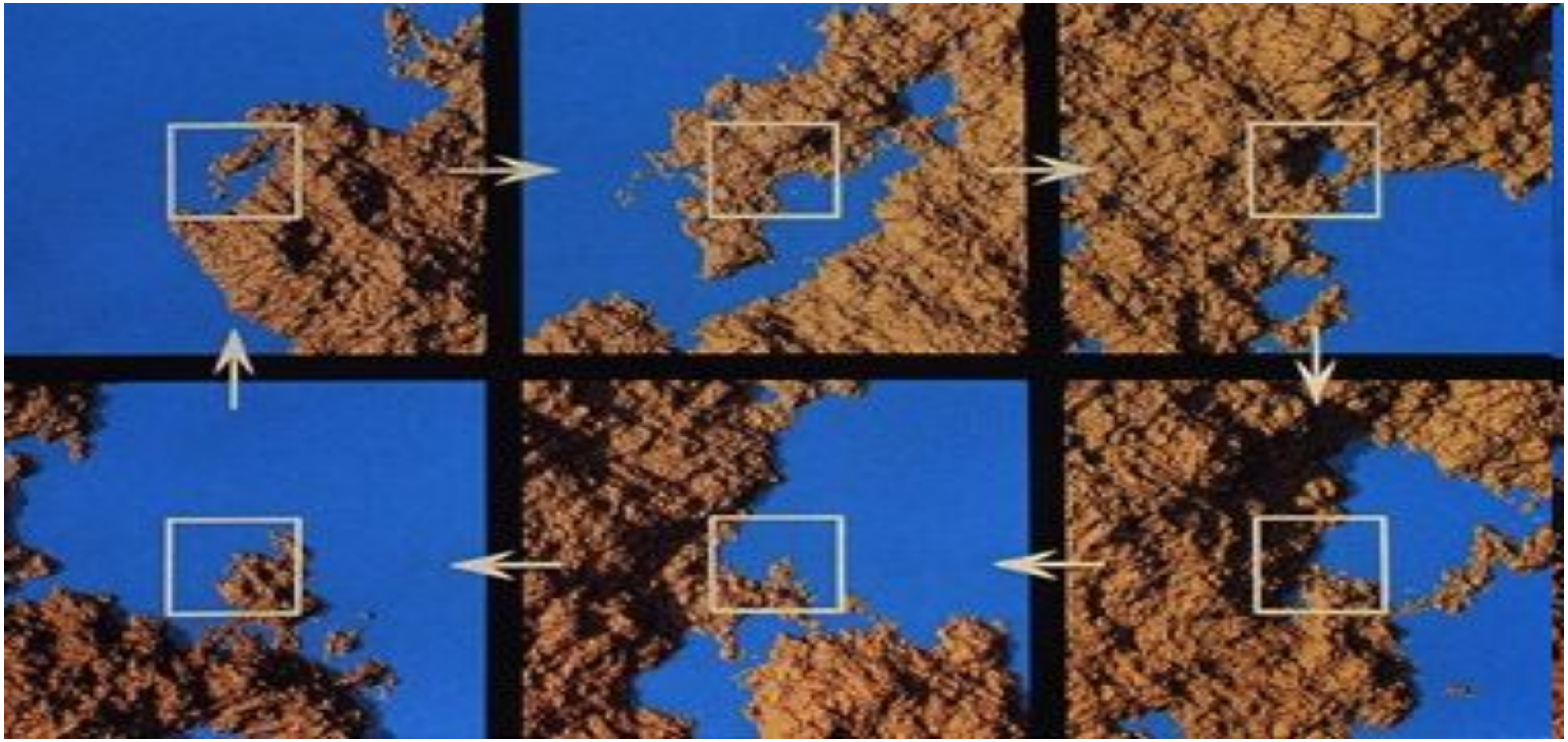
Are these images similar?



Some similarity?



Coast line (artificial)



Some mathematics

- $2^3 = 8$ $\log_2 8 = 3$
- $2^4 = 16$ $\log_2 16 = 4$
- $2^{10} = 1024$ $\log_2 1024 = 10$

- $2^x = y$ $\log_2 y = x$

More mathematics

$0 < q < 1$:

$1 + q + q^2 + q^3 + \ldots + q^n + \ldots = 1/(1-q)$

$1 + q + q^2 + q^3 + \ldots + q^n + \ldots = S$

$1 + q + q^2 + q^3 + \ldots + q^n + \ldots =$

$= 1 + q(1 + q + q^2 + q^3 + \ldots + q^n + \ldots)$

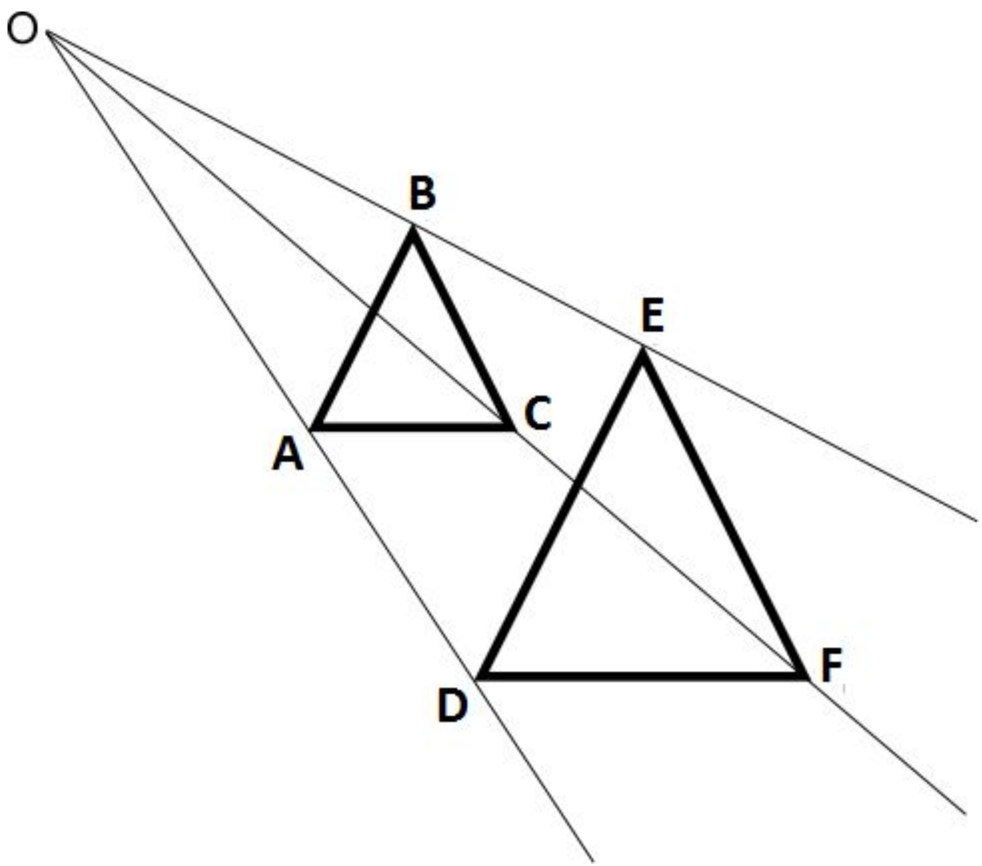
$S = 1 + qS$

$S – qS = 1$

$S(1-q) = 1$

$S = 1/(1-q)$

Homothety



ABC □ DEF
k = OE/OB
(about 3/2)

DEF □ ABC
k = OB/OE
(about 2/3)

k > 1 – enlargement
k < 1 – shrinking

The dimension

M – a set

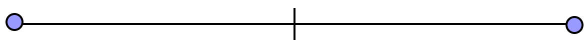
$f_1, f_2, \ldots f_p$ – homotheties with coefficient k

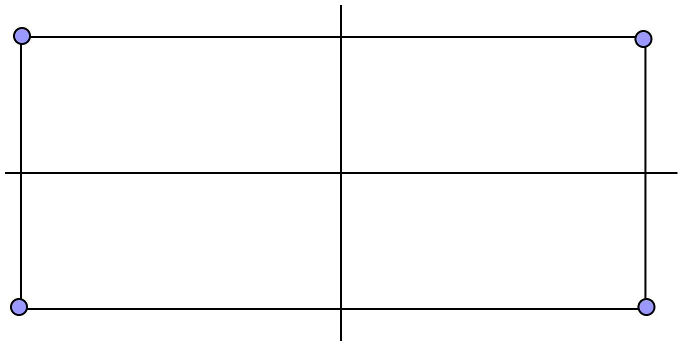
$M = \cup f_p(M)$

- Dimension of M: $d = \log p / \log (1/k)$

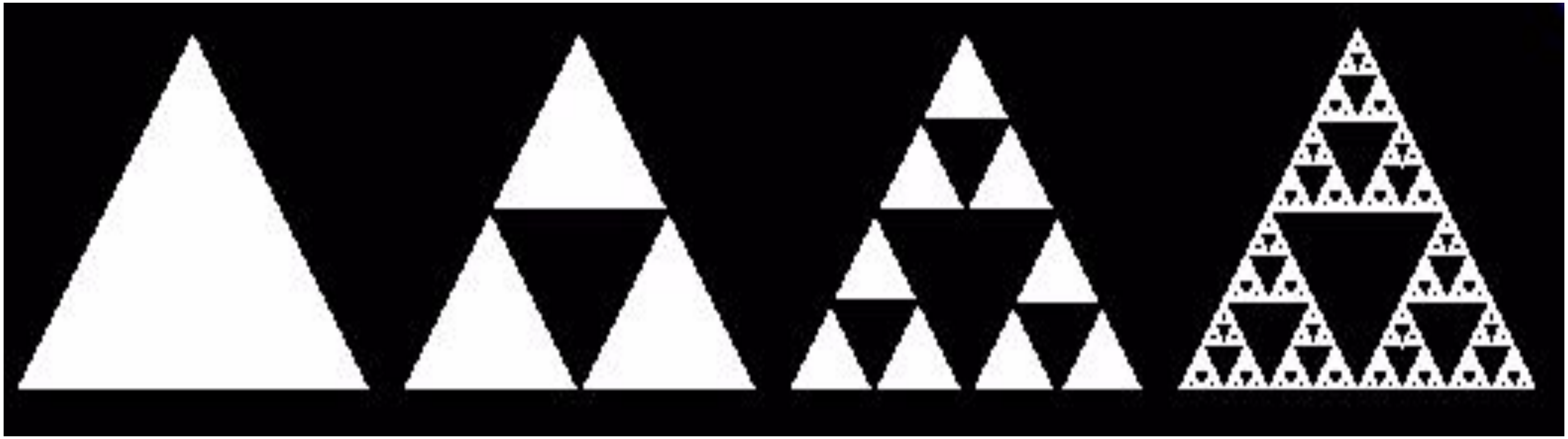
Simple cases

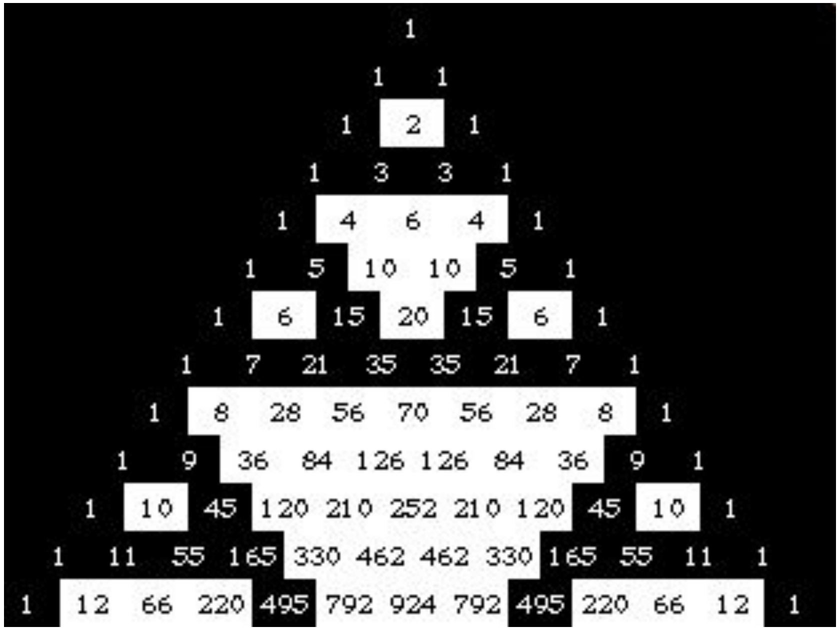
- a line segment
- p = 2, k = 1/2
- the dimension is d = log 2 / log 2 = 1

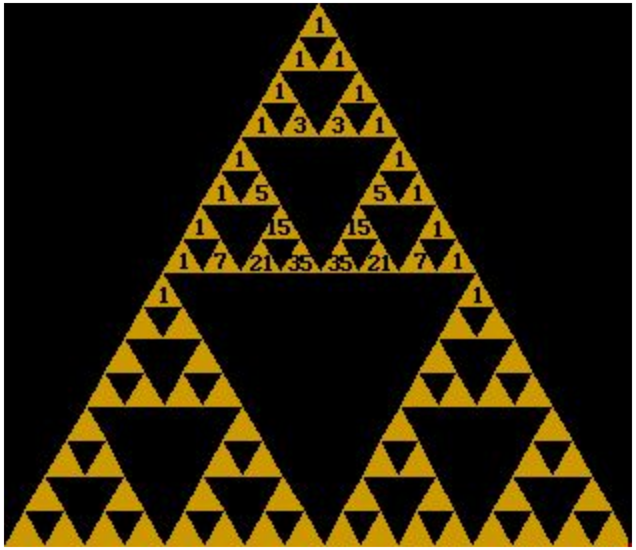




- a rectangle
- p = 4, k = 1/2
- the dimension is d = log 4 / log 2 = 2

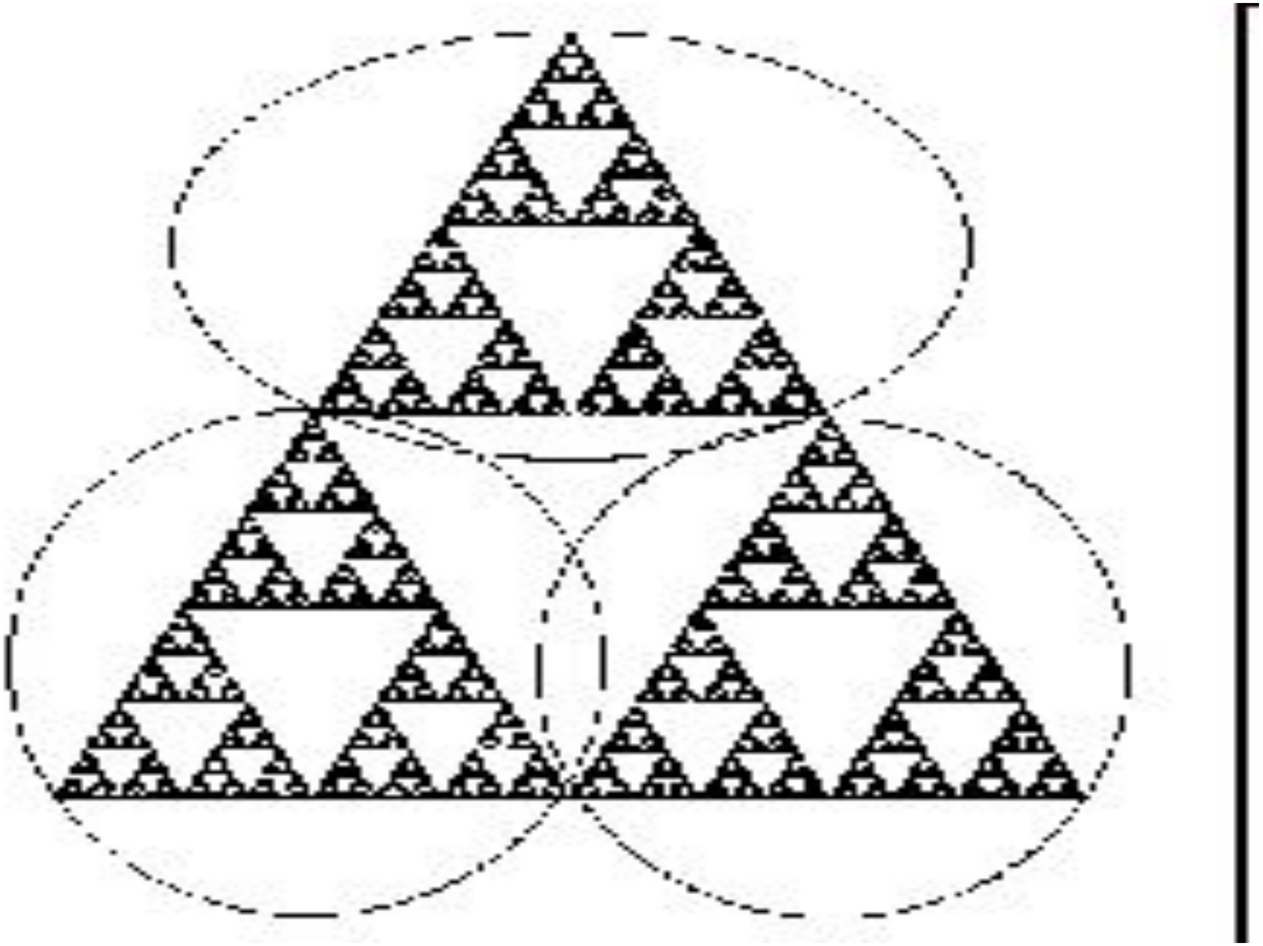



1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1



The Sierpinski triangle

inside



The area of the Sierpinski triangle

- The area of the original triangle: 1
- Step 1: ¼ cut away
- Step 2: 3*(¼ * ¼) cut away
- Step 3: 3*3*(¼ * ¼ * ¼) cut away

………. etc

- $\frac{1}{4} + 3*(\frac{1}{4} * \frac{1}{4}) + 3*3*(\frac{1}{4} * \frac{1}{4} * \frac{1}{4}) + \ldots =$
  $= \frac{1}{4} [1 + 3*(\frac{1}{4}) + 3*3*(\frac{1}{4} * \frac{1}{4}) + \ldots] =$
  $= \frac{1}{4}*[1+(3/4)+(3/4)^2+\ldots]= \frac{1}{4} * 1/(1- \frac{3}{4}) = 1$

Zero area remaining??????

- Is the Sierpinski triangle a line?
- What is the difference between an are and a line?

DIMENSION!!!!

Dimension of the Sierpinski triangle

- $p = 3$
- $k = 1/2$
- $p = 2, k = 1/2$
- the dimension is

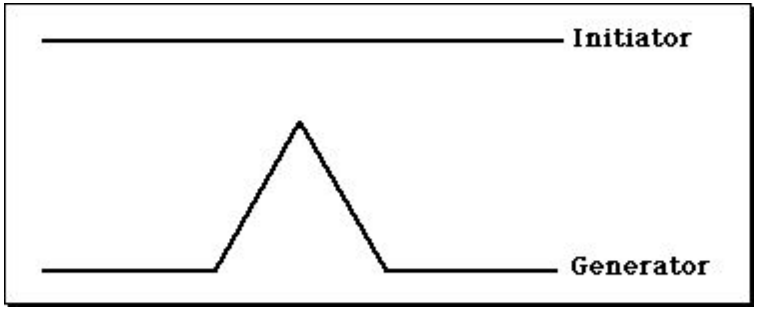
$$d = \log 3 / \log 2 \cong 1.58$$

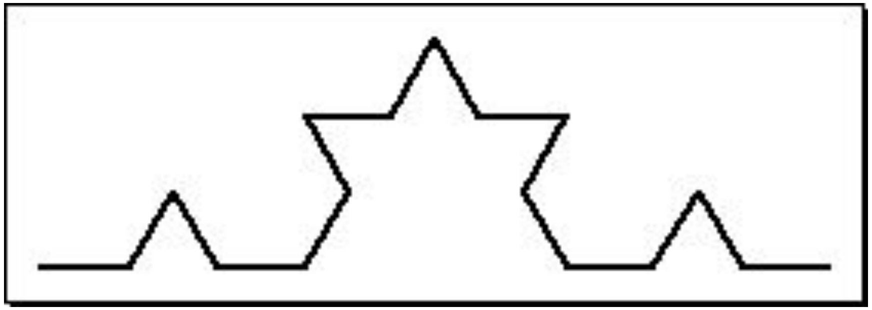
Fractals

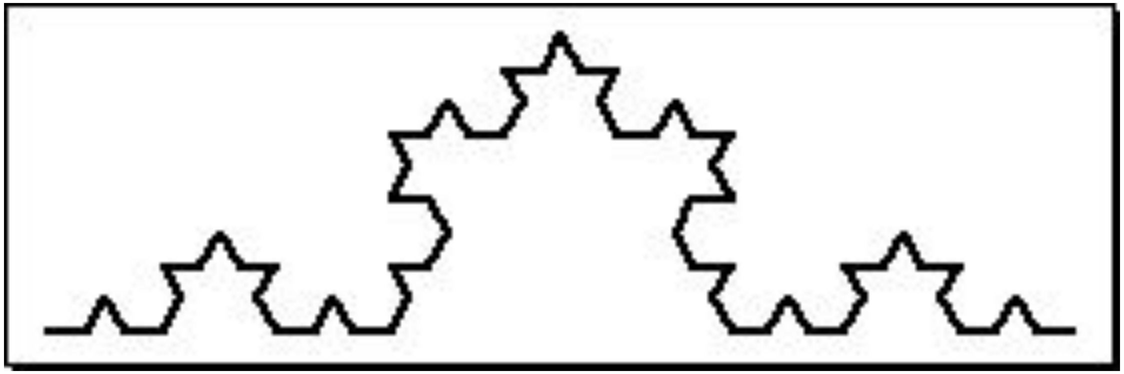
- sets for which their dimension is NOT an integer
- typical property – a part is similar to the whole set (self-similarity)
- 1975

Repeating patterns

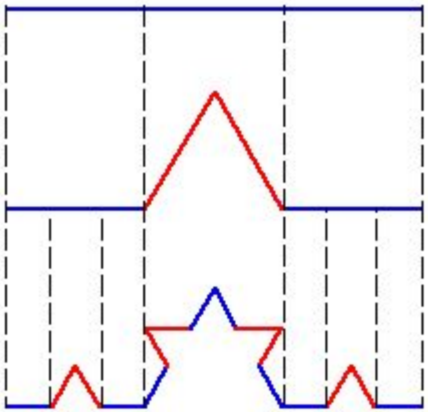
- Koch curve

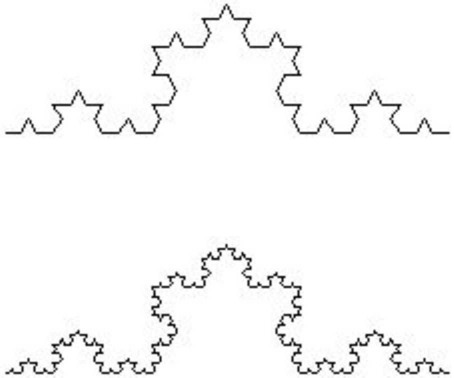






The length of the Koch curve





0	1	2	3	4
1	4/3	4*(4/(3*3))=16/9	$4*4*(4/(3*3*3))=4^3 / 3^3$	$4^4 / 3^4$

length in the n-th step: $(4/3)^n$

The Koch curve

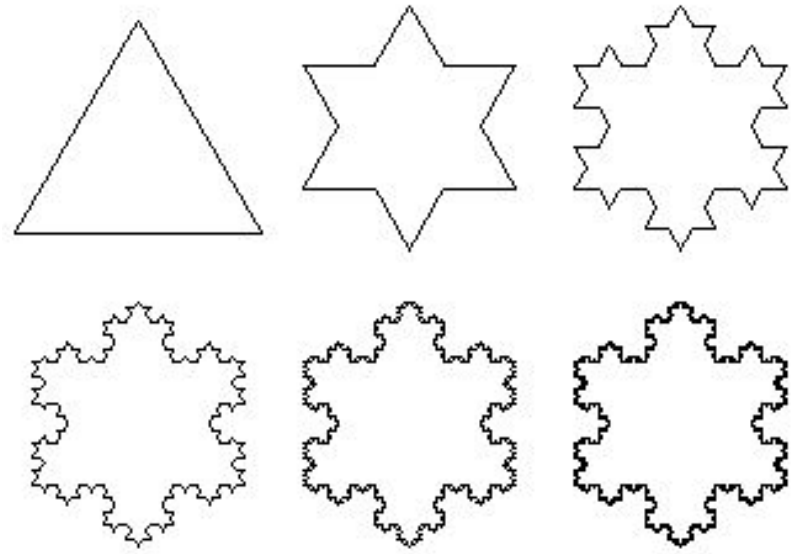
- it is infinitely long
- each its part is infinitely long
- has not a tangent line (at no its point)
- dimension ≅ 1.261

Fractal elements in the art

- https://www.youtube.com/watch?v=rRgXUFnfKIY

The Koch snowflake

- infinitely long
- bounds a finite area



The last piece of mathematics

- $z_0 = 0, z_{n+1} = z_n^2 + c$

$c = 1$:

$z_0 = 0, z_1 = 1, z_2 = 2, z_3 = 5, z_4 = 26, \ldots$

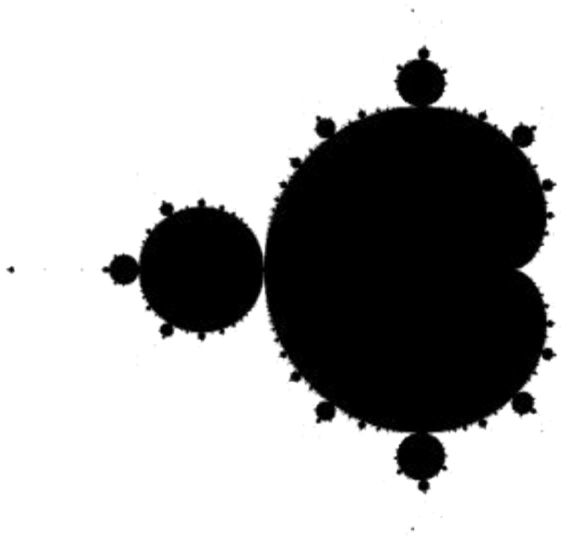
$c = -0.5$

$z_0 = 0, z_1 = -0.5, z_2 = -0.25, z_3 = -0.4375, z_4 = -0.308, \ldots$

For some c the sequence goes to infinity, for some not.

The Mandelbrot set

- The set of those c (in the complex plane) for which the sequence does NOT go to infinity



The Mandelbrot set

http://www.youtube.com/watch?v=0jGaio87u3A