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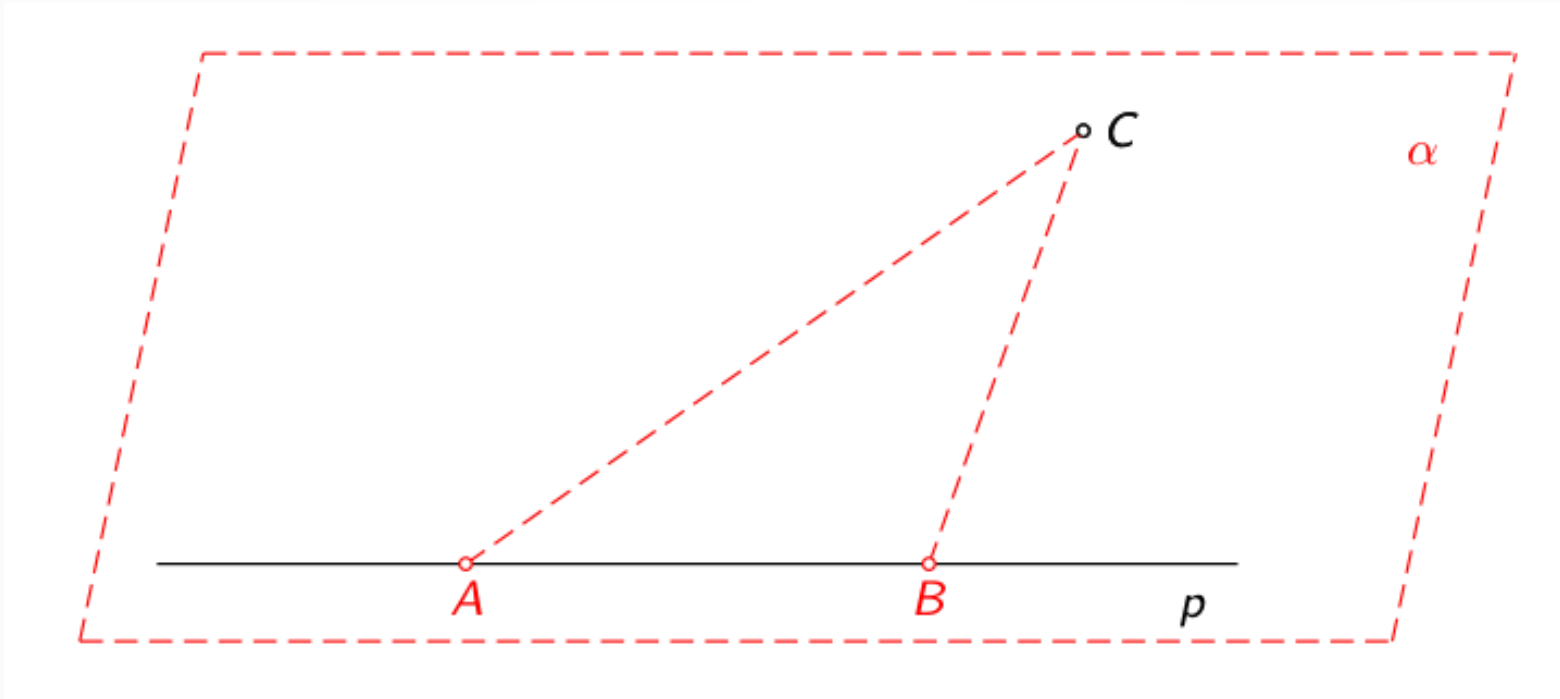


Niš, September, 2022

Theorem prover Isabelle in the teaching of natural and mathematical sciences

Dr. Sana Stojanović Đurđević

Theorem proving – example



Theorem: If a point C does not belong to a line p , then there exists a plane such that the point C and the line p lie on that plane.

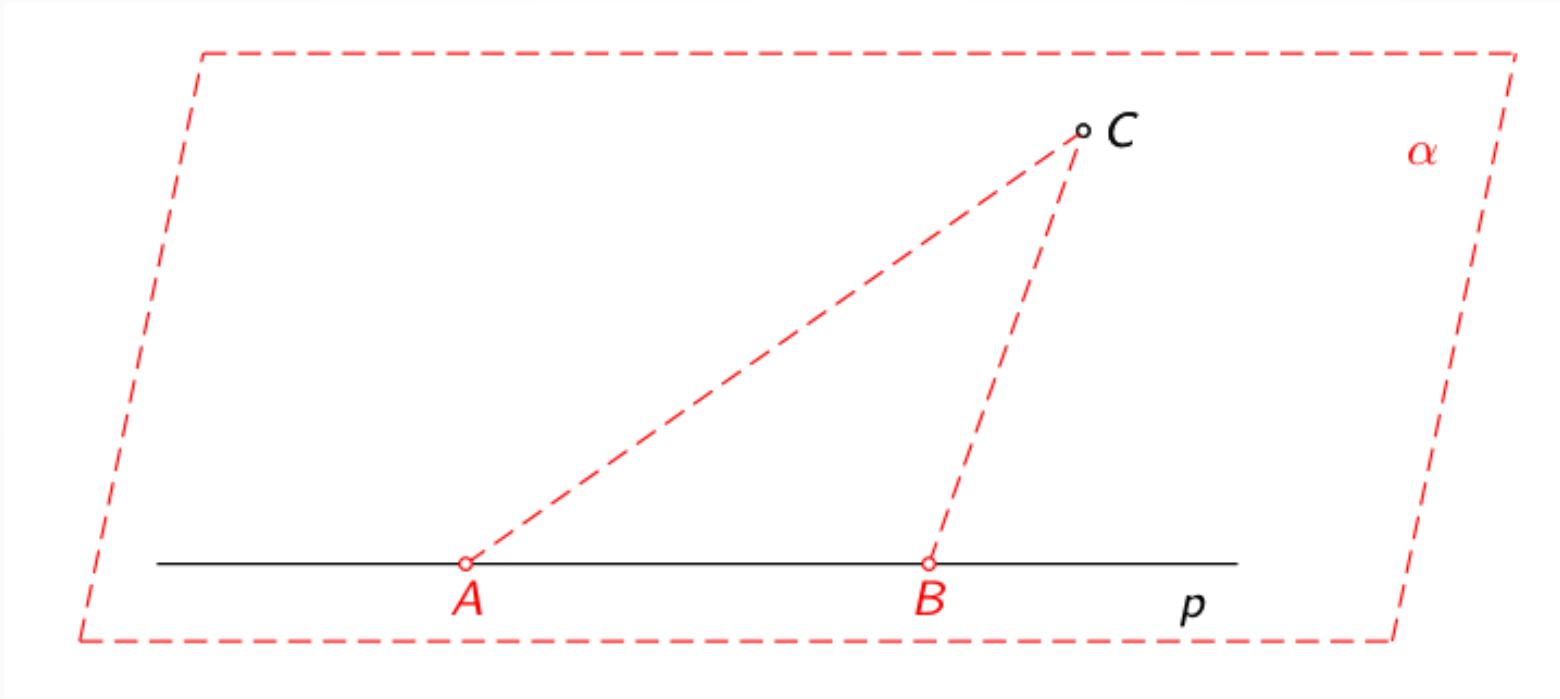
D1 The line p contains at least two points A and B

D2 Points A , B and C are non-collinear

D3 There exists a plane α that contains non-collinear points A , B i C

D4 Line p lies on the plane α

Example – first step



Theorem: If a point C does not belong to a line p , then there exists a plane such that the point C and the line p lie on that plane.

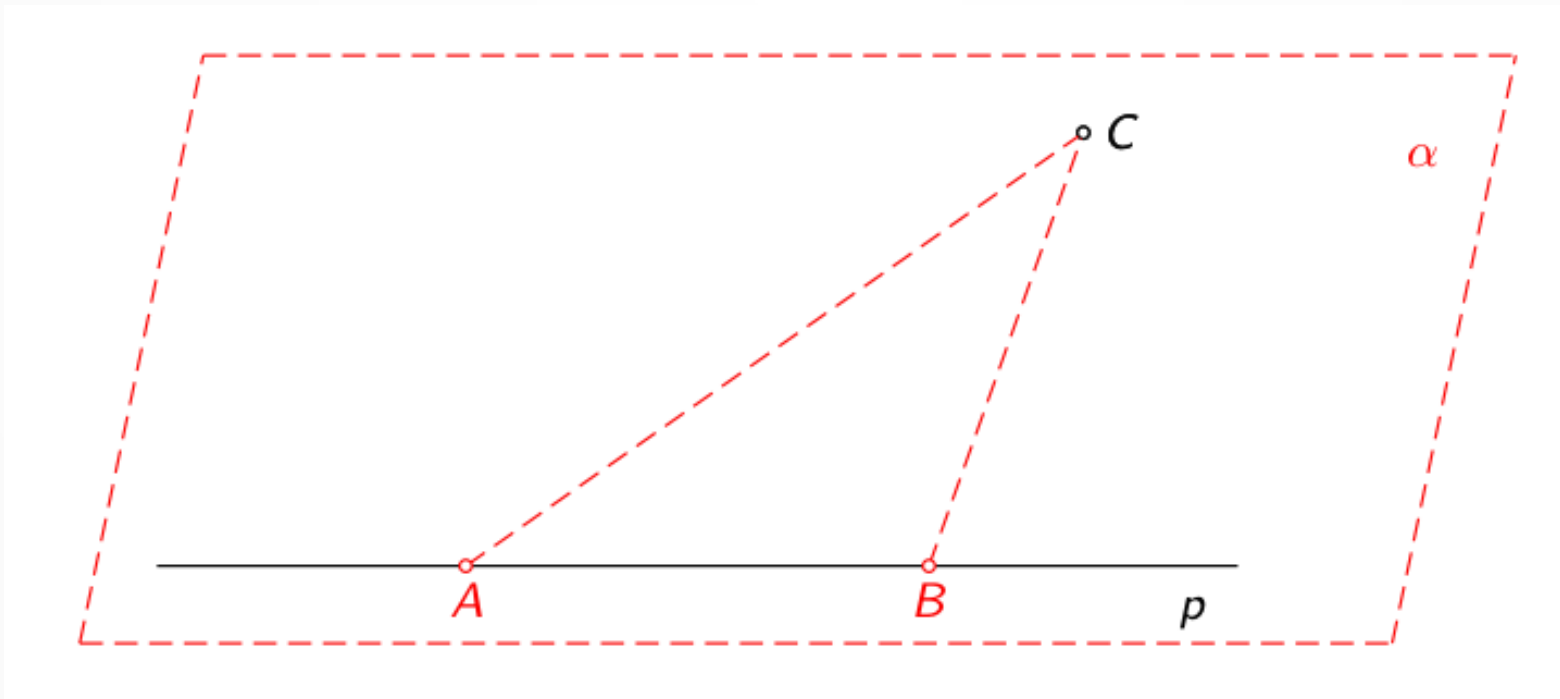
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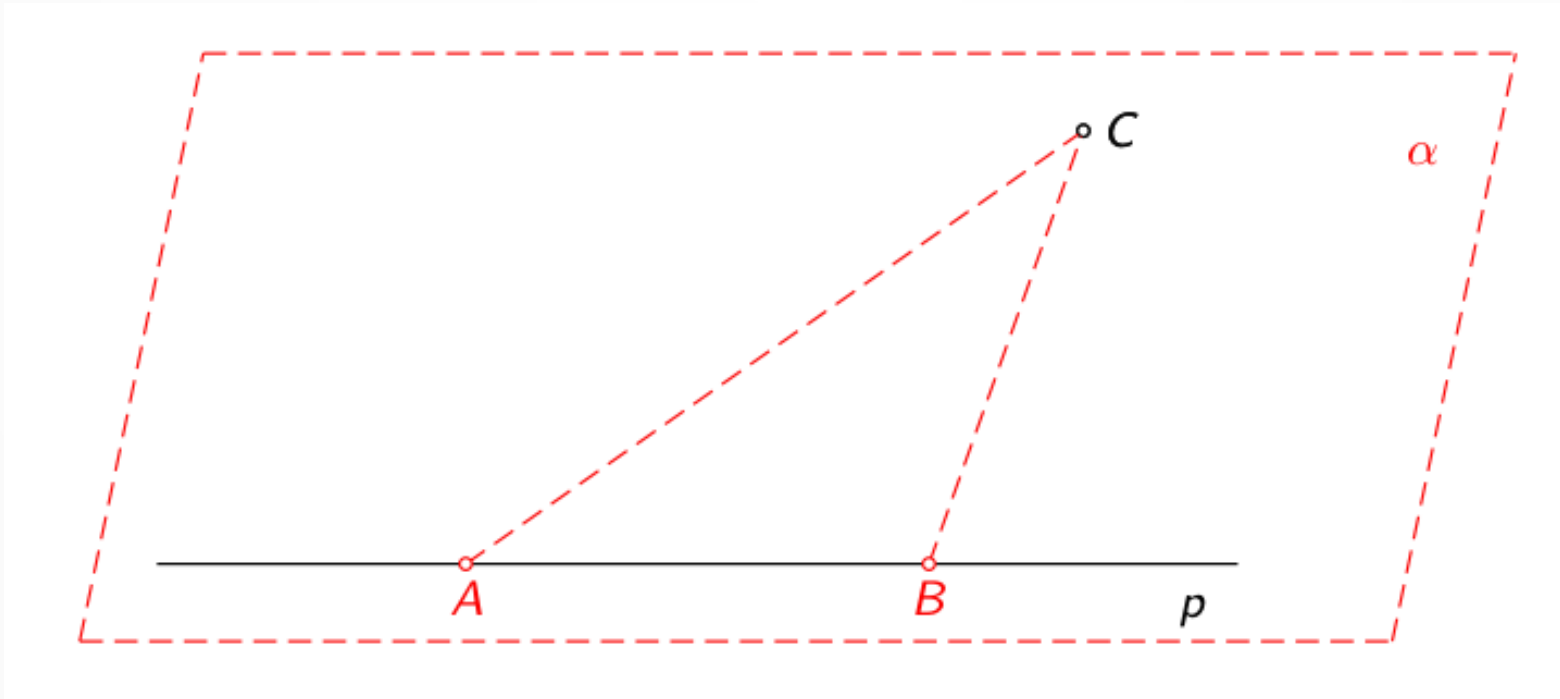
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Example – third step



Theorem: If a point C does not belong to a line p , then there exists a plane such that the point C and the line p lie on that plane.

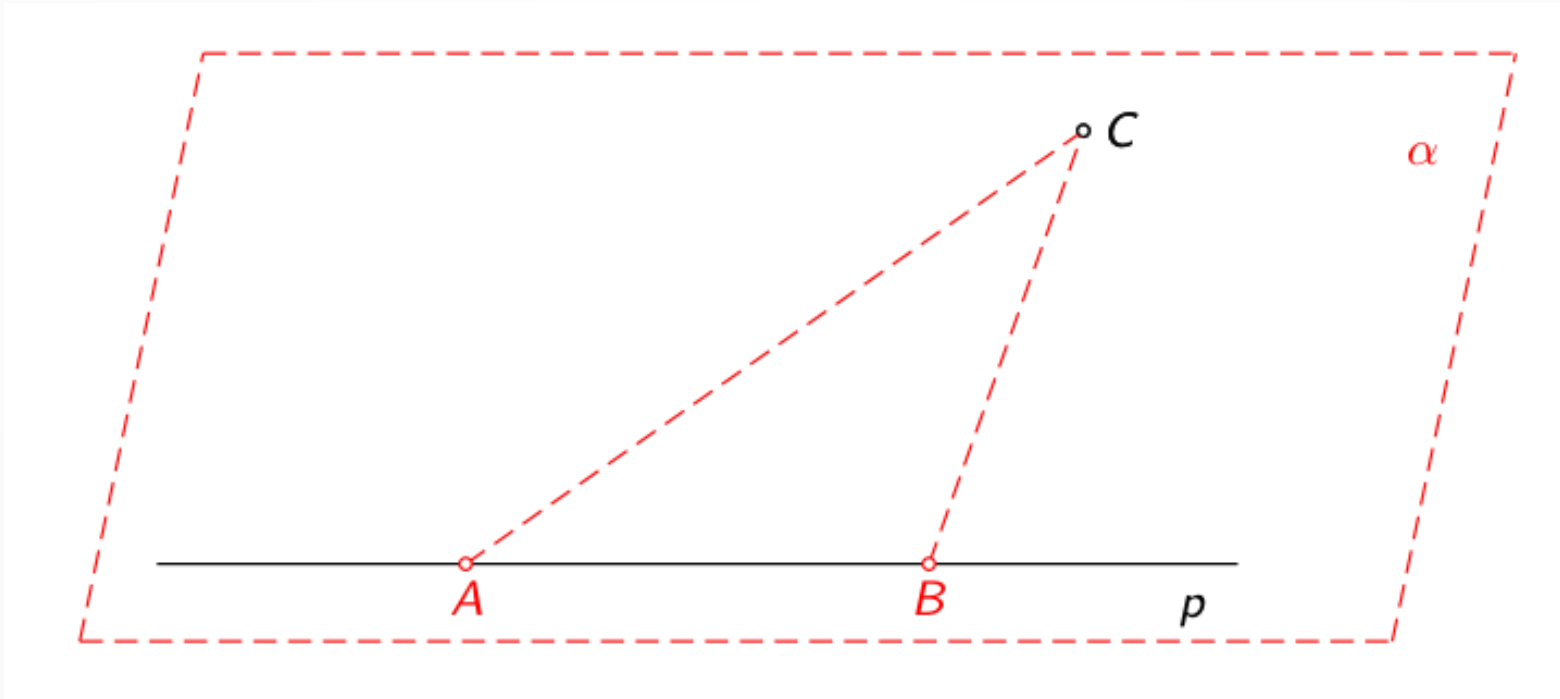
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Example – fourth step



Theorem: If a point C does not belong to a line p , then there exists a plane such that the point C and the line p lie on that plane.

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Theorem proving

- Theorem proof
 - A logical argument that given statement is a consequence of the axioms, using inference rules of a deductive system
- Two main contexts
 - Educational - proofs in mathematical textbooks (pen-and-paper traditional proofs):
the emphasis is on the idea, short but understandable proofs
 - Scientific - Formal proofs (programs for interactive theorem proving):
the emphasis is on correctness of the proof
- A proof explains why the theorem is true, and a formal proof does so in great detail.

Interactive theorem proving

- Theorem proving
 - Automated theorem proving
 - Interactive theorem proving
- Interactive theorem prover Isabelle
 - Two levels of writing interactive proofs proofs:
 - Script proofs (apply rule)
 - Isar proofs (proof, assume, show,...)
 - Very rich notation, close to classical mathematics
 - Support for functional programming and formal software verification

Interactive theorem proving using Isabelle

More information:
www.matf.bg.ac.rs/~sana/uidt.htm
www.matf.bg.ac.rs/~filip/idt

Textbook

Introduction to interactive theorem proving
Sana Stojanović Đurđević, Filip Marić

Mathematics

Remind yourself of the basics of mathematical logics
Work with sets and functions, natural and real numbers

Interactive files

The book chapters are created as individual Isabelle files
An unique feature which enables direct access to all the codes and materials

Programming

Automatic generation of verified code in Haskell, Scala and ML
Work with lists, trees, define new data types, use basic algorithms



- Course “Professional Development in Educational Interaction and Communication”
 - Gent University
 - Belgrade and Korce
- Theme 3 - Poster
 - A forgotten learning and communication approach

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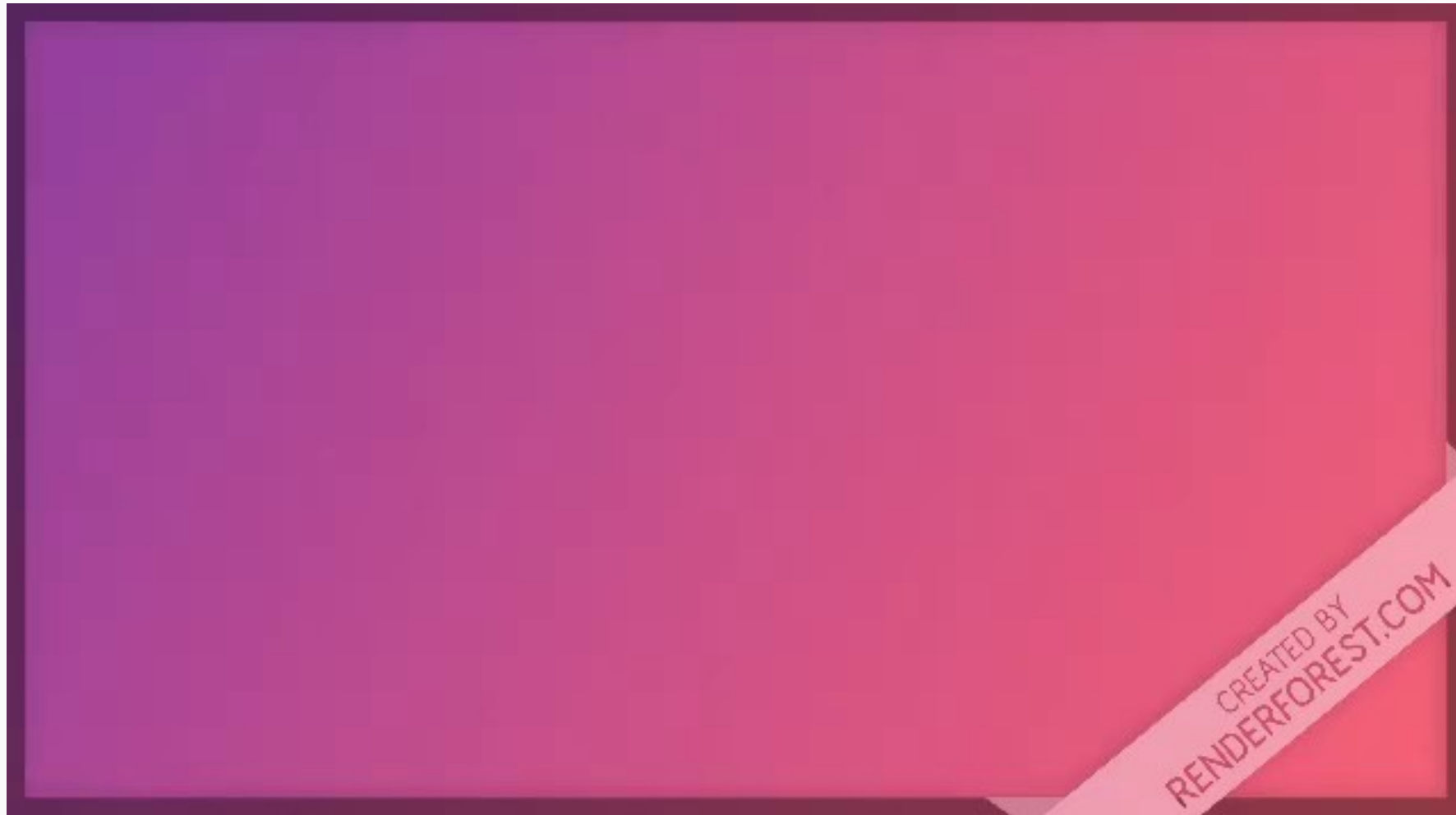
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Theme 5 – Animations as an interaction and communication strategy

url: http://poincare.matf.bg.ac.rs/~sana//uidt/Isabelle_animation.mp4



Introduction to interactive theorem proving

- IV year of bachelors studies – Informatics
- Detailed documentation
 - Isabelle files, exercises, video materials
 - Textbook - PDF file (333 pages) in Serbian only
- Individual work (Problems from International Mathematical Olympiad Short List with Solutions)
 - Github repository (over 100 different problems formalized - with solution, and over 150 problems formalized - without solution)

Solutions

- Whole material is created directly in Isabelle
 - Individually created theories (individually checked) that correspond to different chapters
 - Every Isabelle file that is independent from others
 - Student has a unique opportunity to test the material directly
 - Easy to test, change and master it step by step
- <http://poincare.matf.bg.ac.rs/~sana//uidt/uidt.pdf>