

Introduction to the fuzzy set theory: motivation, examples, discussion

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Motivation-Why fuzzy sets?

- ▶ Fuzzy sets are one of the **main tools of fuzzy logic**.
- ▶ Fuzzy logic is a mathematical discipline that obtains popularity at the turn of the eighties and nineties for fascinating applications carried out in Japan and later in other countries.
- ▶ The most successful applications they are in **control and regulation**, but there are applications in image recognition, classification, decision making and other areas.

Motivation-Application

- ▶ The main source of success is that **fuzzy logic includes vagueness** and works with the meanings of expressions of natural language.
- ▶ Fuzzy logic has **several applications**: it was applied by companies, Boeing, NASA, Sony, Siemens, and others.

Motivation

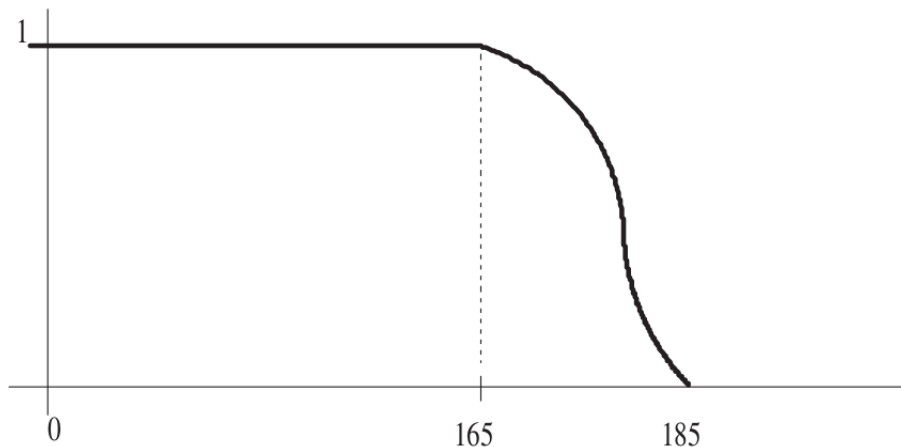
- ▶ A fuzzy set **generalizes the classical notion of a set**. The motivation is based on the following idea.
- ▶ Consider an example of how to specify a set of *large people*.
- ▶ We agree that large people belong to the interval $[160, 220]$ cm.
- ▶ We are not able to specify large people **exactly**. WHY?????
- ▶ For example, if we decide that a large person is at least 175 cm tall, “And what height 174.6 cm?”

Motivation

- ▶ Consider the set of all heights $[40, 220]$ cm which is called **universum**.
- ▶ Assign a number from the interval $[0, 1]$ to the height of the above considered universum, which will express the **truth degree** of the claim that a given height indicates a “ large man.”

Demonstration

- ▶ Let us assume the universum $[40, 220]$ which represents the set of all heights.



Discussion about fuzzy set

Question

What do fuzzy sets look like that represent **small** people, **small** houses, **small** salaries.

Answer

EQUALLY

Question

What needs to be changed?

Answer

Universum

Discussion about fuzzy set

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Answer

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What needs to be changed?

Answer

Universum

Demonstration

- ▶ We define first the set U called the **universe of discourse** or briefly the universe.
- ▶ It can be a set of elements of **any kind**. For example: set of plants, people, dogs, etc.
- ▶ Very often a certain set of numbers. This case is particularly important for fuzzy regulation, where
 - ▶ “deviation”, “change deviation” or “action” are used.

Fuzzy set

Zadeh's definition (1965)

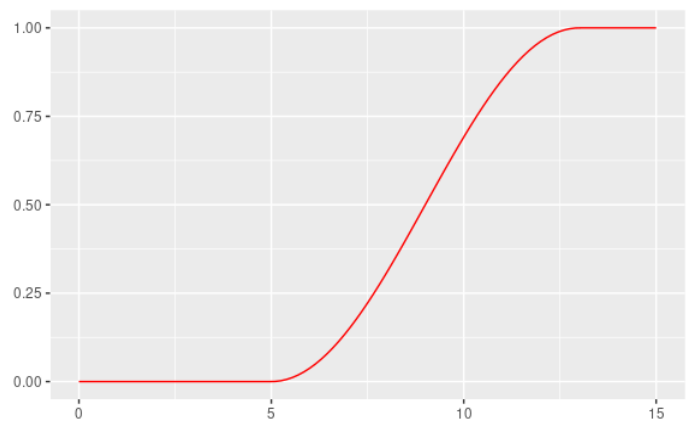
Fuzzy set from the mathematical point of view is the function

$$A : U \longrightarrow [0, 1].$$

We can say that the fuzzy set consists of elements x from the set U , $x \in U$, such that each of which is assigned a number $a \in [0, 1]$ called the **membership degree** of the element x to the fuzzy set A .

Scientific result-an example of fuzzy set

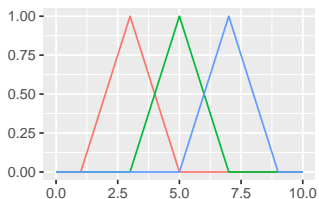
Example of fuzzy set of excellent salary in EUR



Scientific result-an example of fuzzy set

Example of triangular fuzzy set

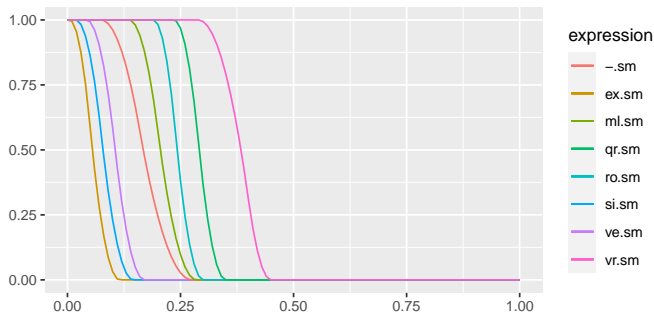
- ▶ This group of fuzzy sets are used for the representation of fuzzy number (“around three”, “around five”)



Scientific result-an example of fuzzy set

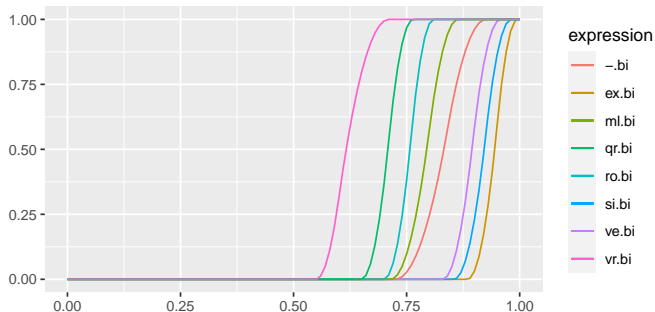
Example of evaluative linguistic expressions based on SMALL

- ▶ These fuzzy sets are used for the interpretation of evaluative linguistic expressions.



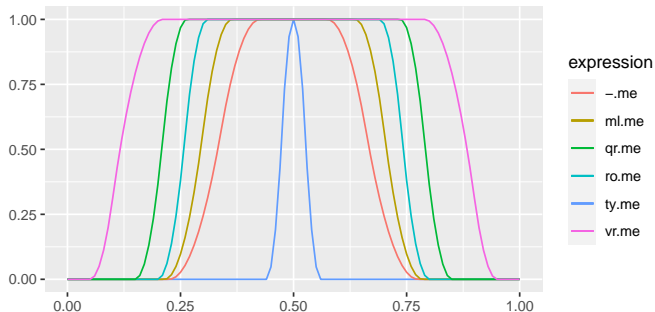
Scientific result-an example of fuzzy set

Example of evaluative linguistic expressions based on BIG



Scientific result-an example of fuzzy set

Example of all the evaluative linguistic expressions



Example of fuzzy set

Let us assume universum $U = \{0, 1, 2, 3, \dots, 10\}$. Then

- ▶ $A = \{0.4/1, 0.9/2, 0.5/3, \dots\}$

Operation on fuzzy sets

- ▶ **generalization of classical operation:** intersection, union,
- ▶ **Łukasiewicz operations:** Łukasiewicz union, Łukasiewicz intersection, residuation, etc.

Example of union of fuzzy sets

Union

Union of the fuzzy sets A and B is the fuzzy set C which is defined as follows:

$$C = A \cup B \quad \text{iff} \quad C(x) = A(x) \vee B(x) \quad (1)$$

Example

- ▶ Let $A = \{0.4/1, 0.9/2, 0.5/3, \dots\}$ and
 $B = \{0.5/1, 0.7/2, 0.8/3, \dots\}$
- ▶ Then $C = \{0.5/1, 0.9/2, 0.8/3, \dots\}$

Example of intersection of fuzzy sets

Intersection

Intersection of the fuzzy sets A and B is the fuzzy set C which is defined as follows:

$$C = A \cap B \quad \text{iff} \quad C(x) = A(x) \wedge B(x) \quad (2)$$

Example

- ▶ Let $A = \{0.4/1, 0.9/2, 0.5/3, \dots\}$ and
 $B = \{0.5/1, 0.7/2, 0.8/3, \dots\}$
- ▶ Then $C = \{0.4/1, 0.7/2, 0.5/3, \dots\}$

Application areas

- ▶ regulation and decision making (LFCLC software-second presentation)
- ▶ fuzzy approximation of function using F-transform (it was developed by prof. Perfilieva)
- ▶ linguistic summarization
- ▶ linguistic interpretation of natural data
- ▶ understanding of human reasoning (fuzzy logical syllogisms)
- ▶ linguistic characterization of times series
- ▶ image processing

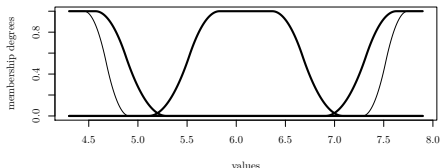
Linguistic interpretation of natural data

- ▶ Each numeric column of the original data set was using **fuzzy GUHA** method transformed into a degree of a membership to the fuzzy sets: **small** (Sm), **very small** (VeSm), **medium** (Me), **big** (Bi), **very big** (VeBi)

Original Data Set:

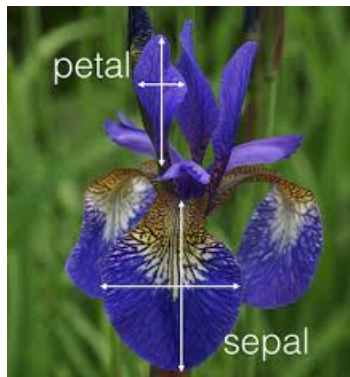
	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
1	5.10	3.50	1.40	0.20	setosa
2	4.90	3.00	1.40	0.20	setosa
3	4.70	3.20	1.30	0.20	setosa
4	4.60	3.10	1.50	0.20	setosa
5	5.00	3.60	1.40	0.20	setosa
6	5.40	3.90	1.70	0.40	setosa

Linguistic Expressions for Sepal Length

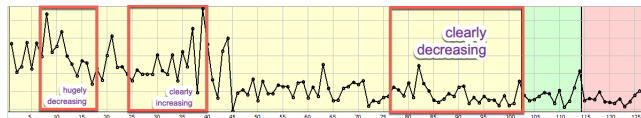


Scientific result

Most irises with both sepal and petal small length have not its petal width small.



Linguistic characterization of time series



- ▶ Most(many, few) analyzed time series stagnated recently but their future trend is slightly increasing.
- ▶ Most(many, few) analyzed time series stagnated recently but their expected trend is slightly increasing.

Understanding of human reasoning

Many diseases which **are not** lethal are virus diseases.

All virus diseases **can not be** cured by antibiotics.

Some diseases which **can not be** cured by antibiotics
are not lethal diseases.

Many animals which **are not** mammals are fish.

All dolphins are mammals.

Some animals which **are** not dolphins are fish.

Conclusion and discussion

- ▶ In general, it is very important that the teaching is dynamic, interspersed with demonstrative examples.
- ▶ It is also important to supplement the lessons with demonstrative pictures that will enable students to understand the issue.
- ▶ If the course is supplemented by independent work of students, this work will help to better prepare for the final exam and will also develop communication between the student and the teacher.