Strengthening teaching competencies

Digital competences

Project acronym:	**TeComp**
Project full title:	**Strengthening Teaching Competences in Higher Education in Natural and Mathematical Sciences**
Project No:	598434-EPP-1-2018-1-RS-EPPKA2-CBHE-JP
Number of grant contracts	2018-2467/001-001
Web address of project	www.tecomp.ni.ac.rs
Funding Scheme:	Erasmus+
Coordinator Institution:	University of Nis
Coordinator:	PhD Jelena Ignjatović
Project duration:	15.11.2018 – 14.11.2022
Work package:	WP2 – Activity 2.4. Guidelines for technological enhancement of teaching and learning
Lead organization of WP7:	University of Nis
Version of the document:	v.02
Status:	Final
Dissemination level:	Internal

DOCUMENT CONTROL SHEET

Title of document	Strengthening teaching competencies – digital competences
Work Package	WP2 – Upgrading educational infrastructure at the PC HEIs
Last version date	1.11.2021
Status	Final
Document version	v.02
File name	Final version IT_English.docx
Number of pages	142
Dissemination Level	Internal

CONTENTS

PREFACE

These guidelines and instructions for wider integration of ICT in teaching and learning are prepared within the Erasmus+ project "Strengthening Teaching Competences in Higher Education in Natural and Mathematical Sciences - TeComp".

The TeComp project is envisaged as a regional-scale project in Serbia and Albania whose clear goal is to improve the quality of teaching and learning, in the field of natural and mathematical sciences at the partner country universities, in line with advance EU practices.

Technology can enable new approaches as to how learning is delivered and assessed and can make certain pedagogic approaches viable in higher education.

The broad set of technology applications, which can enable learner choice means that this training material considers a wide range of issues: topics oriented to blended learning, with choice given to the learner about when and where they learn, opportunities being made for personalized learning with the student finding their own pathway through learning material, and support for a wide range of devices and systems so that learners can choose their preferred application or platform.

The publication deals with digital competences of teachers at higher education institutions that are needed for successful education process. In this regard, the selection of topics is carefully performed in order to develop and increase digital competences of teachers at higher education institutions but also to integrate contemporary pedagogical principles.

Principles and guidelines in setting efficient and beneficial Virtual Learning Environment that are presented in this publication are intended to help teachers and students utilise several new teaching tools available for teaching in natural sciences and mathematics.

Author:
Jelena Ignjatović

QUESTIONS:

What are the basic concepts of flexible pedagogy?

What are the advantages of combined compared to the traditional way of learning?

What are the opportunities and challenges of a flexible approach to e-learning?

What is an inverted classroom?

How to design educational interactive material well?

How to involve students in the process of creating educational material?

How to assess the quality of teaching content?

What is the significance of feedback?

Creating interactive teaching materials
Flexible pedagogies; Blended learning; Flipped classroom

Contents

Introduction

       We can say with certainty that we live in the digital age, surrounded by the latest technological advances. The rate of technological change is constantly increasing and does not show any significant stagnation. Technology has caused massive changes in the economy, in the way of communication and interconnection between people and, of course, in the way of teaching and learning. As our educational institutions are built mostly for a different era than the one we live in, teachers are faced with the great challenge of the need for change. How can we ensure that we develop generations of students who are capable of facing an increasingly unstable, uncertain and complex future? What should be kept from the current teaching methods and?

The goal of the TeComp project is for higher education institutions in Serbia and Albania to join the process of teaching and learning transformation, which is actively taking place at the universities of developed European countries. All change processes involve risks, but also represent opportunities for modernization and improvement. Numerous activities need to be implemented in order to adapt our education system to the European Higher Education Area (EHEA). We are facing far-reaching reforms that include profound changes of a structural, functional and substantive nature that have a direct impact on institutional and professional culture.

The adoption of a new educational model aimed at acquiring skills necessarily includes a different evaluation and implementation of teaching, as well as defining proposals for teacher training and their professional development within lifelong learning.

Competency-based education (flexible learning) involves changes in planning, methodology, assessment systems or teaching that have been offered to students in the past, so it is difficult to imagine them without proper teacher training.

It is expected that this teaching material will contribute to the development of teaching in the field of pedagogical-psychological sciences and the methodology of teaching natural and mathematical sciences. Through the trainings, teachers will present a general framework and specific examples of the application of modern methods and technologies in natural sciences and mathematics. The main goals are:
(a) to introduce training participants to new tools that they will be able to implement and use for different courses at university level
(b) that trainers share their experience in the above-mentioned fields with training participants and demonstrate ways to improve the quality of higher education used on the European University scene.

Flexible learning

–Flexible pedagogy and electronic learning

We will focus, in further work, on how e-learning, known as technologically advanced learning, can support flexible pedagogy and cover a range of topics that explain how technology can enable new choices for students.

Flexible learning focuses on giving students a choice in the pace, place, and mode of their learning, and all three aspects can be developed and promoted through appropriate pedagogical practice, a practice that itself can be supported and improved through e-learning.

E-learning deals with the use of computer technologies to support learning, whether it is learning locally (on campus) or remotely (at home or at work). Technology can enable new approaches to the way teaching and assessment of learning and can make certain pedagogical approaches sustainable and adaptable when it comes to higher education.

A wide range of technological applications that enable different methods of teaching give teachers and students a plethora of choice, so the material considers a wide range of issues:

- an adoption of combined learning, with the choice of the place and time of learning made by the student;

- the ability to presonalise the learning experience by allowing the student to choose the way they traverse the learning material;

- support for a wide range of devices and systems, so the students may choose any platform they wish.

While learning technologies provide new opportunities, they can also create dilemmas for institutions, with new questions about shared learning, plagiarism, and the choice of resources to enable their use. From an institutional perspective, e-learning can offer new opportunities for higher education to support flexible pedagogy, with the potential to help realize high quality teaching with great impact and significant research.

Let us consider the connections between e-learning and pedagogy and the new possibilities that this opens up.

Flexible learning deals with the pace, place and mode of learning:
- the pace usually focuses on different knowledge delivery schedules, which can be shortened, accelerated or slowed down, either within complete programs (for example a three-year undergraduate degree) or by enabling students to work at an individual pace;

- the place entails a physical location, which may be formed at work or at home, or on a road;

- the regime encompasses learning technologies and combines or distance learning.

For example, in the context of technologically advanced learning, technology can support flexible methods, with the possibility of accessing teaching materials outside the prescribed schedules, which allows for a flexible pace.

According to the above, the pace, place and way of working are variables that allow the pedagogical approach to be placed in a three-dimensional space of flexible learning. In the following diagram, the lower, front left point has no flexibility along either axis, and the level of flexibility (choice) increases, while in space it moves from left to right, front to back, and bottom to top.
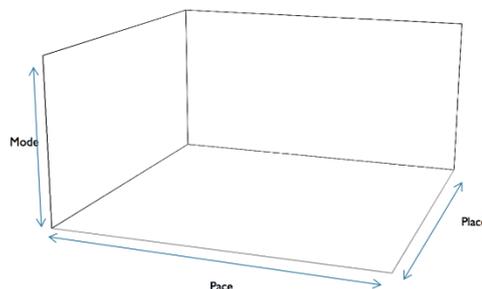
Taking into account the various forms of teaching, learning and assessment that are enabled by the development of modern technologies, we present some forms of modern methods of teaching and learning:
- team projects and group work; adaptive / flexible assessment through various online tests; the assessment of knowledge changes depending on the progress and achievement of students

within the test, and the test can ask more complex questions to students who are progressing well;

- providing information and changes in timely feedback using IT.
- use of technology to enhance learning and teaching, e.g. focusing on content that generate users and interactions between users through blogs, forums, social networks, etc .;
- providing information and changes in timely feedback using IT.
- These concepts require the application of information and communication technologies in teaching and learning, which includes:
- Computer-based learning / training, which includes forms of teaching that emphasize the use of computers as a platform for performing activities;
- computer-supplied learning materials;
- m-learning, as a form of e-learning where the delivery platform is a mobile device - e.g. laptop, smartphone or tablet;
- virtual learning environments: portals that provide access to learning support, including course information, communication (forums, messaging, announcements), course content (notes and lecture sources) and assessment and feedback;
- models (usually 3D ones) where participants can explore and learn in a simulated virtual environment;
- computer assessment / e-assessment, which uses computer technologies to assess students.
- open learning, by which learning resources are shared through open licenses and agreements, e.g. Massive Open Online Courses (MOOC);
- technologies for cooperation and communication.

–Opportunities and challenges of a flexible approach to e-learning

It is evident that new technology offers potential flexibility in what is learned, how it is learned and where it is learned, as well as numerous ways to access resources and information and interact with colleagues and between students and teachers. However, such flexibility can create potential confusion, especially about deciding what, where and how to study. It can also create information overload - due to too many resources to handle and too many teaching materials. As the place of study now becomes a choice - in an institution, at home, at work or on the move - the student faces the new challenge of choosing the right location, understanding what to do and doing the necessary work.

Teachers have an opportunity to plan work with different students (individualization of teaching), to provide a wide range of materials adapted to different learning styles and contexts, using different tools and new media and ways of interaction and communication. However, difficulties now arise for teachers: how to identify, select and adopt pedagogical practices that are enhanced by the application of information technologies, that provide flexibility and improve the goals of teaching, and how to develop their own skills for their use.

New opportunities open up for higher education institutions to include more students and new potentials for sharing resources with other institutions, but they also face obstacles on how to develop

quality processes and support systems for planning and widespread application of flexible learning and how to deal with behavior and student requirements under the influence of these changes.

Thus, the basic questions that need to be addressed when developing modern pedagogy with the use of e-learning arise:

1. How much can students prepare for the nature of flexible e-learning, especially when / if the focus of control shifts from professor to student?

2. To what extent can teachers be helped to manage a wide range of technologies and resources, and more importantly in developing an approach to teaching that involves their effective use?

3. How can institutions cope with the diversity and flexibility needed to support new ways of teaching and learning?

Combined learning model

Combined learning aims to combine the use of information technology and traditional forms of teaching. The rise of virtual learning environments (VLEs) as a ubiquitous platform for hosting and delivering supporting materials means that, for most students, some minimal experience with combined learning is probably already the norm. However, where the VLE is used exclusively to store copies of lecture notes and slides, this can be considered a more flexible and accessible library, as students can access course materials at any time and from anywhere, but of no other importance. In terms of designing effective teaching and learning activities, the provision of lecture materials can lead to students either not attending or not being involved in lectures. A small change of approach to avoid this, for example leaving gaps in the lecture notes to be filled in, can be effective and can shift the course towards a combined approach, in which the traditional way of working prevails.

Combined learning can be more effective when some lecture or workshop-style activities are replaced by online material - either a media clip or other interesting content - and this is developed through traditional seminars or some form of discussion or assessment through a computer. This flipped classroom style offers the advantage that the approach to teaching and learning is student-focused. In terms of flexibility, students must be able to access equivalent content exclusively through online material, which is a challenge for institutions where campus experience is a key part of the educational experience.

The advantages of combined learning are:

- Improving traditional lecture courses through independent work, with flexibility provided in some modules within the programme;
- delivery of materials electronically with limited requirements for real-time interaction;
- distance learning with a minimum amount of real-time requirements.

"When applied successfully, combined learning has the following characteristics:

Personalisation: providing a unique learning guide for students

Agency: providing opportunities for students to participate in key decisions in their experiential learning

10

Authentic results: providing opportunities for students to create concrete work results both locally and globally

Connectivity: providing students with the opportunity to learn experientially through mutual cooperation or cooperation with experts in the field

Creativity: providing individual and collaborative opportunities for students to create projects and thus build skills for their future."

Flipped classroom

According to the above, it is clear that combined learning can simply be defined as a teaching and learning activity that combines face-to-face physical activities with learning that uses new information and communication technologies. Flipped, flipped or flipped classroom is an element of combined learning that integrates both face-to-face learning during the class through group discussion, and distance learning outside the class, following asynchronous video lessons and online collaboration. (Heilesen, 2010; Lean, Moiser, & Newbury, 2014; Poon, 2014).

In recent years, the flipped classroom has become one of the new technologies in education and is slowly becoming the standard of teaching and learning, as it encourages the active learning of students in higher education.

Classroom reversal teaching is an approach to teaching and learning activities in which students watch a video lesson outside the classroom through distance learning and have practical activities live in the classroom. A flipped classroom is also known as a student-centered approach to learning, which means that students are more active than teachers in the classroom. In this case, the professor can be considered a "facilitator" (not a "tutor") with the task of motivating, directing, and giving feedback on student work. Therefore, by applying this approach to teaching and learning activities, the teacher can change the traditional video lecture that students can listen to anywhere outside the classroom. The flipped classroom allows students to watch the video at the desired time and multiple times as needed, so they can learn at their own pace. This type of activity also increases students' cooperation in distance learning outside the classroom. So, by turning the class, students do not spend as much time listening to long lectures in the classroom, and they have more time to solve problems individually or in cooperation through distance learning with peers. This approach to teaching and learning also contributes to a better understanding of the use of technology in teaching and learning activities; students will use different technological media in independent learning, while the lecturer will use different technological media in their teaching practices.

The flipped classroom is based on the theory of the revised Bloom's taxonomy of the cognitive domain of knowledge. This taxonomy provides six levels of learning. The explanation is arranged from the lowest to the highest level:

1. Memory: At this stage, students try to recognize and recall the information they have received and try to understand the basic concepts and principles of the content they have learned.

2. Understanding: Students try to show their understanding, interpret information and summarize what they have learned.

3. Application: Students practice what they have learned or apply the acquired knowledge to real situations.

4. Analising: Students use their critical thinking to solve problems, lead debates with friends, compare answers with peers and make a summary. Students acquire new knowledge and ideas after building critical thinking or debate in group activities. At this level, by learning, students produce creative thinking.

5. Grading: Assessment or audit of knowledge is a significant result that is achieved through relationships. At this stage, students self-assess or mutually evaluate overall learning concepts and can make judgments about how successfully they have learned.

6. Creation: Students are able to independently design, construct and produce something new by applying the acquired knowledge (Bloom, 1969).

In the concept of a flipped classroom, remembering and understanding, as the lowest level of cognitive knowledge, are practiced outside of lecture hours (Krathvohl & Anderson, 2010). While in the classroom, students focused on multiple forms of cognitive work, including application, analysis, assessment, and creation. The following figure illustrates the level of student learning in flipped learning according to Bloom's revised taxonomy.
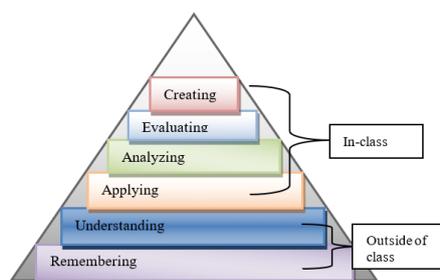
In the flipped model, lower levels of knowledge are presented before classes through recorded lectures and video. Reading materials, simulations and other materials also provide basic learning support so that time in live teaching can be spent working at higher levels of learning from application to evaluation.

In the concept of a flipped classroom, students move from the lowest level (remembering) to achieve the highest level (creation). Lankford (2013) noted that a focused classroom focuses on how to support students in reaching a higher level of taxonomy. In addition, Nederveld and Berge (2015) added that in reverse learning, classroom activities are aimed at achieving a higher level of learning rather than listening to lectures and other lower-level thought tasks.

Interpretation of „tutoring"

In the context of a flipped classroom, therefore, a perspective has been adopted that does not consider the lecturer to be a teacher-"tutor", but puts the student in the place of a key participant in education. The student becomes a "tutor" and takes the lead. This means that, in addition to a course designed to put a teacher at the center and which is based on lectures, workshops etc., there is a part of time dedicated to peer teaching. Teachers decide which goals will be achieved by this approach in

teaching: e.g. developing active research competencies, developing communication competencies, developing critical thinking competencies.

Teaching is realized by tutoring groups, while the size of the group varies depending on the level of study. Usually, the groups are larger in undergraduate studies and decrease with increasing levels of study. Tutors are trained and supervised by a teacher. This also allows for regular internal audits to record course-level tutoring activities. Internal auditing allows tutors to share experiences and contributes to improving teaching skills. Tutors work regularly with students (eg once or twice a week). Their activities encourage group cooperation and strive to develop cooperation among students and exchange feedback. We will now present the tasks that the teacher can put before the tutor, to realize the concept of an inverted classroom and learn new tools that contribute to improving teaching competencies and teaching quality.

Educational interaction and comunication

−Knowledge clips

The terminology used in educational videos varies depending on the purpose. After studying this theme, you master the following learning objectives:

- Choosing to develop knowledge clips is based on the choice oflearning objectives.
- Be able to assess an existing knowledge clip on the basis oftechnical and functional criteria.
- Design and implement your own knowledge clip andimmediately indicate its strengths and weaknesses.

Especially due to the Covid 19 pandemic, video clips have veryquickly become a key ingredient of online higher education. A varietyof types of video uses are observed:

- streaming video
- full lesson recordings
- knowledge clips

Today we specifically address the latter category: 'knowledge clips'.They have a specific purpose and that is why they are worked out ina specific way. The label 'knowledge clip' immediately indicates what their goal is: trying to achieve a specific learning objective briefly and to the point. The word 'short' "should be taken literally. The guidelineis that a knowledge clip lasts a maximum of 10 minutes. The startingpoint is a very well-defined learning objective. Anyone who wants to develop a knowledge clip therefore first and foremost aligns this learning objective well, collects all information, supporting materials,writes a script and gets started with video recording software.

Design guidelines for knowledge clips

What do we learn from empirical research about videoclips in terms of designguidelines?

The literature and research focusing on video clips / knowledge clipsin instructional settings usually reports positive effects. But it is goodto emphasize that those effects are not always positive. Muchdepends on the integration of the video clips in the broader context ofthe instructional process.

Preradovic and colleagues (2020) therefore emphasize two important design features that are related to thisemphasis onto integration: "preview the task" and "strengthen thedemonstration with practice". So, it is recommended that beforeviewing the video clip/knowledge clip, there are guarantees thatstudents master the prior knowledge that is presupposed whenviewing the clip. Second, a clip cannot and should not stand alone.After watching the video clip, students get to work building on theknowledge they have experienced are have been introduced to.

Research about video clips is not a discussion about yes / no. Theresearch mainly helps to understand why the use of video has aneffect and which design decisions have the strongest effect. Forinspiration we give a recent study in which the most popularinstructional videos - found on Youtube - were analyzed.

Ten Hove and Van der Meij set up a study in which 250 popular video clips were analyzed. this research is about knowledge clips related to declarative knowledge. So, this is knowledge that is conceptual, factual and not related to procedures, approaches, processes. The research results indicate that the most popular clips exhibit the following:

- The clip's technical resolution is high. Students see things insufficient detail, the image is sharp. Of course, a higher resolution will burden the download speed of the video clip, but developers usually assume a minimum resolution of High Definition (nowadays referred to as HD 1080)

- Static images are added to the video clip. it may sound contradictory but adding a picture to the video just-in-time (spatial and temporal contiguity) enhances the learning effect. The researchers distinguish two types of static additions: iconic pictures (schematic pictures, structures derived from a real image. But also pictures of reality. Second, there are analytic pictures. These are pictures that represent diagrams, graphs, maps, patterns ... It is a more abstract image that integrates a lot of information.

- Dynamic figures are added. These can be realistic representations of a process course; but in addition these can be animations. Are you talking about e.g., glaciers and their shifting; then it is obvious that you represent the 'shifting' dynamically. You will find other examples e.g., in cell division: mitosis, meiosis etc. these are abstract concept sthat you understand better when you see a dynamic image, after, for example, a static image. Many abstract concepts include processes that a student cannot properly 'imagine' providing images thus reinforces the imagery.

- There is a good variety of titles, sound and voices. Text is thus displayed, and the text is spoken. Spoken text is appreciated, but beware! What is 'said' does not have to be literally displayed as text (redundancy). The extra addition of music (e.g., intro) and sounds (e.g., a woes shh sound when a figure appears); does not appear to contribute to popularity and impact. Be careful with those extra sounds and be sure to be careful with background sounds that do not underscore the focus of the video clip. When adding music then "music should not compete with the spoken message".

- Pace: a striking result is that the pace, tempo in popular videos is quite high.

Moussiades and colleagues (2019), based on a theoretical and empirical analysis come to the following results:

- Be brief yet inclusive.
- Use conversational style.
- Pay attention to the aesthetics of the video.

- Control the rate of speech in regards to fast or slow as it is considered that student engagement increases proportionally to speaking rate.

- Define the target audience.

- Don't overload video frames with text.

- Use narration.

- Provide Images that are uncluttered and simple.

- Synchronize audio and visual messages.

- Support variety.

- Control the pace.

- Follow the signaling principle.

- Provide introductory notes or knowledge background related to the content.

- Organize video in sections.

- Use captions.

In the following study, the emphasis is much more on the acquisition of procedural knowledge; knowledge with regard to processes, approaches, step-by-step plans, procedures etc. This is of course a different kind of sport, because the emphasis is on providing multi-sensory support of more complex knowledge. We know that tackling procedural knowledge implies a great deal of declarative knowledge is sufficiently mastered. How could you –otherwise - take into account specific facts, concepts, relationships,theories in the learning process about procedures. This immediately produces the first draft guideline: is a sufficient level of prior knowledge available and/or is the necessary prior knowledge sufficiently refreshed before the instructional video focuses on the procedural knowledge? Erikson & Erikson (2019) also put forward a lot of practical guidelines; it is striking how they emphasize the core of this type of instructional videos: "how to".

It is striking that the authors rightly emphasize in their research that there are few evidence-based approaches for developing these types of video clips. As a result, a lot of available instructional material is of inferior quality. The authors base their particular approach on available research evidence and integrate this into a broader working framework. They summarize the characteristics of live-actioninstructional video clips as follows:

- Visibility: in other words, has everything been made sufficiently "visible" with the video. First of all, this means that attention waspaid to aspects of light, resolution ... play a role. But even moreimportant is the completeness of a recording. For instance, in pole vaulting, you also have a clear introduction to walking up, holding the pole, etc.

- Privacy: take care! Sometimes, as a developer of the video clip, you run up against the limits of what can be made "visible". For example, if you emphasize the pronunciation of a word in a Spanish pronunciation exercise, you will have to take extra time from avariation in the examples (man and woman sometimes voicing differently).

- Recordability: this refers to the extent to which the entireprocess can be included. This often requires additional material to be added after the actual recording in order to map all aspects of a process, procedure, competence; by eg; adding arrows, frames, text, replaying part of the recording. The researchers immediately emphasize the criterion "Verifiability".This refers to

the authenticity of the experiences shown: the setting, the equipment used, the accessibility for the student to the tools, instruments, materials ... supported between the demonstrated action by an expert, teacher ... and the action performed the student has to perform. pay attention to the position, sequence, feasibility of the actions.

- Comparability: what a student ultimately does, is it in line with what should have been done. Consider, for example, the quality of a melody that is predicted in the instruction video:isn't it too fast? Isn't it played on an instrument that exceeds thequality of a student's instrument too much? Is the pace achievable; was it sufficiently divided into small sections beforethe full melody was expected to be performed?

Privacy and copyright

When you film/record people and they can be recognized or identified: did they give permission? Have parents or guardians consented to the filming of minors? Have you respected the copyright of photos, texts, animations, images. Respecting copyright starts with explicitly mentioning the source and can also go further; e.g., that you have paid for their use. Much depends on how your end product will be distributed and whether or not it will be commercially exploited. For example, if the photos are downloaded from iStock the fees have to be paid for them, in the context of use for educational applications. You can of course be flexible with these rules when tackling the task related to this professional development course, but be careful with the distribution of your end product. When you start distributing or start giving access to the product to other people, you need to follow the applicable regulations. Of course, uploading final product in the context of a course is not a problem, as long as the material is not used outside the context of the course. There are many websites with audiovisual material that you can use copyright free (provided the source is mentioned at the end of the recording).

Students making their own videoclip: an alternative approach

It sounds a bit strange in the context of this theme, but why not ask our students themselves to develop a video clip when tackling a task, assignment, question. Such a video clips will show in a dynamic way what students can do. And you have the advantage that you immediately see the students themselves at work in a reading assignment, an explanation, a music performance. But, this is actually not so strange. If you look at Bloom's taxonomy in terms of learning objectives, you realise that many learning objectives cannot be tested or pursued on paper, or without a strong form of interaction. How can students demonstrate that they can create something (e.g., perform a piece of music, independently develop apiece of evidence for an assignment, elaborate a text) or approach a task at the evaluation level (e.g. conduct an experiment, give an assessment in a work of art, checking a solution of achemical reaction), or proving that they can analyse something (e.g. comparing two economic systems, describing links between climate and economy). Of course, this can be done on paper, but when students work this out with video, you get to see a much more powerful, personal and complete effect. Moreover, we know – from experience - that students prepare, repeat, re-record this several times and ultimately submit their "best" work. The first part of the next lesson was to view the student's "best" recording. In this way, giving feedback takes place in a much more structured, systematic way and with respect for the actual control of the learning objective.We do not elaborate this approach in this theme; but be sure to take this into the teaching practice as a basis for evaluating student learning or as an instructional approach in class.

Tools

We can distinguish between two types of tools:

1. Tools designed for developing instructional videoclips.
2. General video editing tools.

   In this category you will find many professional tools with which you can develop advanced applications. Today, the tools are so user-friendly and there is so much accompanying instructional material available that this is certainly within reach of teachers. For example, one can use Camtasia, others use IVideo, Moviemaker, Pinnacle e.t.c. or one of the tools recommended to you through the many sites on this subject. Sometimes there is even decent software on your smartphone.

   Many students ask the question: but don't you need expensive recording equipment? No, most inexpensive cameras and even a lot of smart phones (resolution HD 1082) are already capable of recording very good quality video. That video recording immediately forms the basis for your end product. Only pay attention to the sound quality, you quickly have noise, wind, background noise. That is why purchasing a simple extra microphone immediately has added value.

−Interactive video

Interactive video contents are very powerful toolsin teaching and learning and they greately facilitated mastering the following learning objectives:

- Provide a substantiated argumentation as to why interaction with video has added value.
- Use available knowledge clip and video clips as a basis for the design of an interactive video clip (focusing on either feedback or on processing skills).
- Assess the quality of an interactive video clip based on evidence-based criteria.

Explore this website about tools that can help you make video use more interactive. You will immediately notice that there are many tools and that the importance of interaction is very strongly emphasized:
https://practicaledtech.com/2020/02/24/how-to-create-interactive-videos-three-options/

Types of interaction in videos

Why "interaction" is so important?

In the previous chapter we explored the development and use of knowledge clips/video clips in an instruction setting. We cited research on the potential positive effects of this form of interaction and communication. But research emphasizes that these positive effects can quickly disappear due to the lack of "interaction". This brings us to the core question: How to promote interaction when passively watching video clips/knowledge clips?

Purpose
To reduce discrepancies between current understandings/performance and a desired goal

The discrepancy can be reduced by:
Students
- Increased effort and employment of more effective strategies *OR*
- Abandoning, blurring, or lowering the goals

Teachers
- Providing appropriate challenging and specific goals
- Assisting students to reach them through effective learning strategies and feedback

Effective feedback answers three questions
Where am I going? (the goals)	Feed Up
How am I going?	Feed Back
Where to next?	Feed Forward

Each feedback question works at four levels:

| **Task level** | **Process level** | **Self-regulation level** | **Self level** |
| How well tasks are understood/performed | The main process needed to understand/perform tasks | Self-monitoring, directing, and regulating of actions | Personal evaluations and affect (usually positive) about the learner |

Research about "what works in education" often falls back on the overview work of Hattie (2009). In his 2015 book on Visible Learning- the role of feedback, Hattie and Clarke (2018) zoom in on therole of feedback. He could already emphasize that whether or not provided with feedback, entails a learning effect ofeffect size 0 .75, that means that 50% of the students perform better after receiving feedback. In addition to teachers' views on the value of feedback, he also asks students what they expect from feedback. It is striking that pupils mainly expect feedback to answer their questions about: "Where to go next?". Feedback today has become part of a growing "feedback culture" in which assessment is not an end point of a learning process, but a living part of teachers' learning and instructional approaches. Based on research, Hattie & Clarke(2018) describe the following characteristics of such a feedbackculture (2015):

- feedback is part of a formative assessment culture;
- tackling mistakes is a 'normal' starting point for learning;
- feedback is focused on the task and not the person;
- the result of giving feedback is not a 'score', but a focus onwhat you can do now, change, tackle
- The emphasis on those characteristics of feedback is not surprisingwhen we consider the criteria that tie in with good feedback.

The characteristics of good feedback are given in the following figure (Hattie & Clarke, 2018)

In our checklist we build further on these 8 characteristics of feedback. What we also learn from the literature is that feedback relates to three big questions:

- What have I achieved so far in function of the set learning objectives?
- Did I achieve the learning goals effectively?
- What can I then tackle and/or how can I adjust things?

Hattie and Timperley (2007) refer to this as feed up, feed back and feed forward.The investigations also shows that the forms of feedback can focus on the task, the process followed, the way in which students monitor themselves (self-regulation) and about their person (self level). Research shows that the two strongest forms of feedback address the task and the process. With feedback on self-regulation, a teach eremphasizes the self-evaluation, planning, reflecting, checking in between, an so on.

The addition of questions, tasks, assignments ... provokes behaviorthat we can classify according to the different behavior levels in the taxonomy. Because this behavior is more than just looking and listening, it is predictable that something else can be learned as well.You will see that in the checklist for this assignment we explicitlyrefer to Bloom's taxonomy to check whether the questions /comments elicit sufficient strength in cognitive processing. If you want more examples of adequate "verbs" that fit in with the taxonomy, just type 'Bloom Taxonomy' in your search engine and you will find plenty that might better fit your typical knowledge domain. You might add a search term for your domain: math, physics, history, economy e.t.c.

BLOOM'S TAXONOMY DIGITAL PLANNING VERBS					
REMEMBERING	UNDERSTANDING	APPLYING	ANALYZING	EVALUATING	CREATING
Copying	Annotating	Acting out	Calculating	Arguing	Blogging
Defining	Tweeting	Articulate	Categorizing	Validating	Building
Finding	Associating	Reenact	Breaking Down	Testing	Animating
Locating	Tagging	Loading	Correlating	Scoring	Adapting
Quoting	Summarizing	Choosing	Deconstructing	Assessing	Collaborating
Listening	Relating	Determining	Linking	Criticizing	Composing
Googling	Categorizing	Displaying	Mashing	Commenting	Directing
Repeating	Paraphrasing	Judging	Mind-Mapping	Debating	Devising
Retrieving	Predicting	Executing	Organizing	Defending	Podcasting
Outlining	Comparing	Examining	Appraising	Detecting	Wiki Building
Highlighting	Contrasting	Implementing	Advertising	Experimenting	Writing
Memorizing	Commenting	Sketching	Dividing	Grading	Filming
Networking	Journaling	Experimenting	Deducing	Hypothesizing	Programming
Searching	Interpreting	Hacking	Distinguishing	Measuring	Simulating
Identifying	Grouping	Interviewing	Illustrating	Moderating	Role Playing
Selecting	Inferring	Painting	Questioning	Posting	Solving
Tabulating	Estimating	Preparing	Structuring	Predicting	Mixing
Duplicating	Extending	Playing	Integrating	Rating	Facilitating
Matching	Gathering	Integrating	Attributing	Reflecting	Managing
Bookmarking	Exemplifying	Presenting	Estimating	Reviewing	Negotiating
Bullet-pointing	Expressing	Charting	Explaining	Editorializing	Leading

Guidelines for designing interactive video

There are two approaches for designing interactive video materials:

1. Adding questions to existing knowledge clips/video clips
2. Giving feedback/comments for the sent video

For the adding questions to existing video materials the Edpuzzle tool can be used. The work with this tool can with taking a look at the following application of Edpuzzle, where a free – account can be created. The Edpuzzle interactive video can be only look if someone has his own account. When you explore the video clip, you will notice that the video suddenly stops and you have to answer questions.

How can such an interactive video be made?

The procedure of interactive video constuction is explained in detail in the following video clip: https://www.youtube.com/watch?v=L9h86-UmLyg

In the knowledge clip you learn that you can use, your own videos or existing video clips to develop an interactive video. You can also add different types of questions to a knowledge clip (open questions or multiple choice) and whether or not to give feedback. You can find all instructions online. What we do emphasize is not to create a "class" for the assignment for this theme. You can also simply release an elaborated interactive video "public" (so choose"public links") and then you can copy the link and pass it on toothers. Working with a class is of course interesting when you get started with this in your own school and you offer a series of interactive videos to students. You can then immediately have thestudents' answers entered and, if necessary, retrieve the scores based on the feedback entered. Edpuzzle also includes a testingsystem and that is super handy.

For the purpose of providing a feedback on a video of others Videoant can be used. This toolwas developed by the University of Minnesota and is very successfulin education. You can view an introduction to Videoant via: https://youtu.be/1SOE2aQky2I .

With Videoant the annotations can be added to an available video. You can then save that whole - the original video and the annotations – again as a video clip and, for example, return it to the original maker of the video clip. There are several aspects to be considered when watching a video:

- Giving feedback on pronunciation during an oral presentation

- Giving feedback on the description of a solution

- Giving feedback when demonstrating/performing a work

- Giving feedback during a discussion where a student must convince another student of their claim

It can be noticed that a video with feedback can be created in almost all knowledge domains. To use Videoant you firstcreate a - free - account. When using it you will sometimes receive technical comments (that the tool works best with Chrome and Safari and less well with Firefox).

Effect of adding feedback

For the inspiration to this topic, we would like to refer to a research article in which the addition of video feedback in highereducation is investigated. This can already inspire you when viewing your own training (Mahoney, Macfarlane, & Ajjawi, 2019). A striking angle is that the effects are not only examined for pupils/students, but also for the teacher.

As an example of video-based feedback in higher education, we refer to a study in language teaching (English as a foreign language). Özkul and Ortactepe (2017) found that the video-based

feedback was more effective than traditional forms of feedback. But if you search a little further via scholar.google.com and type in "video feedback" and learners as search terms, you will find even moreexamples.

The same is actually true of adding interaction to video. Hattie (2009) indicates that the use of interactive video is associated with an effectsize of d =0 .54. A recent study from 2019 (see Anderson & Davidson,2019) confirms this meta-analysis in which the researchers compareclassic passive video (what they call receptive video) with interactivevideo viewing. Their research examines the brain functions related to learning. And their conclusions therefore emphasize the differenttypes of learning goals that are achieved with receptive versus interactive video viewing. If someone wants to achieve more with his learning goals than developing an initial knowledge base about new learning content (Bloom would call this memorization andunderstanding), then it is better to add interactively to the viewing ofthe video clips. Only then does he reach behavioral levels in the learning goals with regard to application, analysis, evaluation? This research immediately emphasizes the attention that a teacher must have for the learning goals that he wants to achieve when using this technology.

The approach that we put forward in this theme is also evidence-based: adding questions to a video clip appears to have a clearlearning effect.

Don't forget that by adding targeted questions to a video clip, you can also immediately lay the foundation for a different assessment approach: assessments based on interactive video. You can read this in the conference contribution of Blackstock and colleagues (2018). Click on the author's name and you can immediately read the article online. As a teaser, we give you the table with the estimated learning effects by the students involved:

Question	No Gain	A Little Gain	Moderate Gain	Good Gain	Great Gain
Increased confidence that you understand the material	1%	3%	23%	48%	24%
Increased your learning in this course	1%	7%	16%	41%	36%
Increased your understanding of how ideas from this video relate to ideas encountered in the course	3%	5%	20%	39%	34%
Increase your understanding of how ideas encountered in the course relate to helping people address real world issues	3%	6%	18%	30%	44%
Increased your ability to remember the content covered in the video	3%	8%	14%	38%	38%
Increased your ability to connect key class ideas with other knowledge	4%	5%	24%	38%	28%

Notice that the learning effect relates to different forms of "cognitive processing".

–Educational posters

After studying this theme, you master the following learning objectives:

- Acknowledging the position of posters in 21st century highereducation

- Linking poster design to the learning objectives of your owncourse

- Explaining the key design principles of an educational

- Evaluating the educational value of a poster

Posters are part of the "heritage" of education. Besides the use of a blackboard (with the slate as a precursor), the use of posters goe sback a long way. Sometimes those posters were referred to as wall charts. Nowadays, we often use the word info graphics to refer to these posters. Since the 18th century, these wall charts have beenseen as the solution to make education more "visual" (De Buck,2015). You can see the visualization principle as a precursor to whatcognitivists emphasize as the importance of "multiplerepresentations". Typical wall charts showed historical situations,geographic phenomena, images in medicine, veterinary sciences,technology, mathematics, physics, chemistry e.t.c.

The development of color printing, the introduction of full-colorhandbooks and the use of multimedia, whereby the students themselves can search, download and edit images (static and dynamic) themselves via the Internet. Made that the importance of the classic wall plates has disappeared. They have become museum artifacts. But those "classic" wall charts hide a different kind of use of posters that is less known and underexposed in literature and classroom practice: having students design and develop posters themselves. In this third theme we look at this type of poster use. This does not prevent posters from also having a decorative value and ready-madeposters can also still play a role in learning processes. But in this theme we do not consider that specific role. What we also don't cover in this theme are posters that aim to emphasize a specific idea and where one slogan and image are central. These posters also have a possible "learning" value (promotion of safe traffic, healthy food, reading more, tolerance, non-bullying).
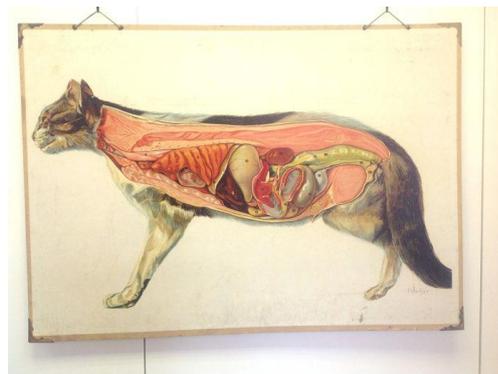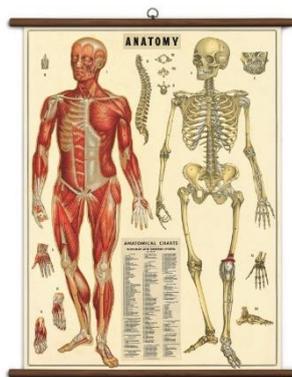
The following article by Stone (1929) contains early examples of how students collect and document chemical elements through a poster (ibid): "How matches are made". With the explanation: "The central picture shows Benjamin Franklin sitting withhis back to the fireplace where the flames of the wood fire aredancing. Below are shown three short verses setting forth that Benjamin had to use flint and steel to start his fire since matches were then unknown, and concluding with the triumphant assertion that ". The common match that you can scratch is quite a marvelous invention. "Above the picture the producers of the poster havemounted a little book describing how they made their matches and the accompanying striking surface. Around the center are arranged tubes containing the different

materials used in match manufactureand there are also shown a number of the matches made by the pupils".

The examples given so far are very close to illustrating settings, situations, events, objects. But there are early examples of use in abstract knowledge where reasoning is central (cause-effect, explanations, comparison). For example, Bell (1928) encouraged herstudents to make posters about how chemical processes can tackle the production of harmful smoke in factories or how chemical waste can be processed, how to fight fire by applying chemical processes or explainthe operation of pesticides in rat extermination. Interestingly, Bell also explicitly states that you can use the poster activity to support the application of chemical knowledge (see Bloom), but that posters can alsobe used to test things out.

Conyers (2003) also emphasizes the role of posters in assessment processes. As an example, she gives the development of posters at the end of a series of lessons on wound care. The students then worked out posters in which they summarize the core of the lessons, substantiate them with sources, illustrate them with photos and drawings. The students could choose which topic they would like to tackle from the series of lessons. A poster session followed after the posters were developed. For the evaluation, emphasis was placed oncriteria such as correct interpretation of information, organization ofthe information, relevance of the images and reference to originals ources. Berry and Houston (1995) apply this in their mathematics lessons where students develop posters in the mathematical modeling lesson. In their research article they list the following learning objective:

- to encourage clear and concise thinking in mathematics;
- to provide opportunities for cooperative learning and discussion;
- to develop learning through peer-tutoring and self, peer and tutor assessment;
- to enhance communication skills;
- to introduce aspects of professional practice.

What is striking is their emphasis on "communication-oriented" learning goals, in a less obvious knowledge domain such as mathematics.

It makes sense to use posters when pursuing creative goals. Hasio(2015) had pupils develop posters that were then hung in the schooland /or class."Overall,the knowledge and skill outcomes reflected a higher level ofprocessing and student learning. Students expressed themselves clearly and logically through their creative posters and demonstrated competence in problem-solving. They were also able to use their concepts to engage in self-reflection through class critiques and recognize that there were social, historical, and theoretical perspectives linked to their creativity. "She also refers to embeddingin a culture: "There is cultural significance for using creative postersin the classroom; like advertising the posters operate on a culturallevel to provide symbolic associations between images, mediafigures, and values related to the student's lifestyle ".

In another example, DeCapua and Marshall "(2019) use English to speakers of other languages in the USA (referred to here as SLIFE:students with limited or interrupted formal Education). The posters are recreated by the students with the aim of: "students create andshare a poster with information comparing and contrasting electoral votes in those two states. As SLIFE are accustomed to sharing responsibility for learning among others, the teacher adapts to the students by making this a pair-work activity. Together, students compare and contrast the two states: their electoral votes, populations, and other facts they choose to research. The final condition for learning that SLIFE are used to — oral instead of written transmission — is also attended to by the teacher because studentspresent their pair-work together on a poster to their classmates. "You notice how the posters play a role in different stages of the learning process. The emphasis is on tracking down informationabout election results, comparing, looking for statements.The posters here fit into a process that is sometimes referred to as "inquiry learning".

In another example, we look again at a less obvious knowledge domain: healthcare. Duchin and Sherwood (1990) had students develop posters that were then used in a poster session. Carter(2012) emphasizes how this aims in scientific settings (conferences, symposia) to create a story and network in 5 to 10 minutes. The focused, summary, condensed, integrated layout of a poster places high demands on its design. And there lies the learning potential that we want to pursue with posters: "Designing a scientificposter is thereof a bit a balancing act. Your challenge is to present asmuch information as possible in as few words a possible using visualaids designed to complement you, yet that can be understoodwithout your presence." Hay and Thomas (1999) had students make posters in biology. In their article they highlight a wide variety of learningo bjectives:

- add variety, new challenges, and diverse assessment to thecourse being taught (Billington, 1997);

- stimulate literature searching experience (Mulnix and Penhale,1997);

- provide opportunities to practise skills in condensinginformation (Sweeney, 1984);

- promote critical thought (Hinzmann, 1996);

- encourage student-staff interaction (see Howenstine etal.,1988; Rush et al., 1995).

The former paragraphs sound a little bit 'dated' and might reflect an old-fashioned perspective. We wanted to reiterate the 'historyt' behindposters, but in th next sections we will emphasize how alive and kicking 'making posters' is as an educational strategy in higher education. At least let you explore

the multitude of tools and apps nowadays available for you and students to develop posters (https://vvv.commonsense.org/education/top-picks/apps-and-vebsites-for-making-posters-and-kolaži).

–Learning objectives to be attained by developing

The examples in the previous section give a first idea of which learning goals can be pursue by having students develop posters:

- brainstorming

- summarizing ideas, texts, books,

- developing a concept map for a complex theme

- description of a troubleshooting approach

- visualization of a process

- timeline for events, historical events,

- table

- venn diagram to map similarities and differences

The list of examples emphasizes here in addition to declarative knowledge (facts, concepts, structures, theories) also strong emphasis can be placed on procedural knowledge (processes, approaches, step-by-step plans). They can be used in all knowledge domains, so posters are quite universally applicable. However, many of the learning objectives will only be achieved whent he posters are used in a broader learning and instruction process, thus involving preparatory activities and the posters being used in individual, group or classroom activities.

Examples of posters in higher education

Check out these examples of posters developed by students inhigher education:

An example from the knowledge domain of 'religion' from Stanford University (https://web.stanford.edu/dept/undergrad/urp/SURP/surpawards04/pages/gabrio.htm).

Example from the biological education domain
(https://librari.leeds.ac.uk/info/1401/academic_skills/ [196/presentations_posters/4](https://librari.leeds.ac.uk/info/1401/academic_skills/196/presentations_posters/4)).

Group poster developed in view of the course Evolutionand Ecology
(https://www.hsph.harvard.edu/public-health-practice- [resources/for-students/resources-for-](https://www.hsph.harvard.edu/public-health-practice-resources/for-students/resources-for-) [practicum/communicating-your-practice-](https://www.hsph.harvard.edu/public-health-practice-resources/for-students/resources-for-practicum/communicating-your-practice-) [experience/student-posters/](https://www.hsph.harvard.edu/public-health-practice-resources/for-students/resources-for-practicum/communicating-your-practice-experience/student-posters/)).

Student academic poster guidelines

The number of lists with guidelines to develop academic andeducational posters is 'endless'. But we give some selected examples.

Hay and Thomas (1999) stress the following poster designguidelines::

Table 1 Five principles of poster production	
Attention-getting	Does the poster make a good first impression? Does it grab the attention of the viewer? This can be achieved through layout, colour, title, and other devices. Getting audience attention is critical to successful communication
Brevity	The poster should make its point(s) concisely
Coherence	An effective poster makes a logical, unified statement requiring no further explanation. It should be intellectually accessible to the intended audience and must be capable of 'standing alone'
Direction	Over-complicated posters discourage and confuse readers. They should be kept simple and focused
Evidence	The argument must be supported by accurate, referenced evidence. The poster must be credible to a critical audience

Berry and Hauston (1995) supplement this with the following criteria in their math posters:

- Is there a logical structure of the poster? and
- Is that "common thread" visually supported?
- Are there any meaningful titles?
- Is the central title attractive, catchy, motivating to read the poster?
- Are the illustrations meaningful (instead of purely decorative)?
- Are they attractive to attract attention?
- Is the problem clearly outlined?
- Is everything sufficiently compact, concisely elaborated?
- Are the illustrations are meaningful and not merely decorative?

–   Is/are there any conclusion(s) provided?:

Duchin and Sherwood (1999) additionally emphasize thelanguage being used:

–   Is the title no longer than 10 words?

–   Does the language use suit the target audience?

–   Is the target audience familiar with the symbols, abbreviations,j argon used?

–   Is all recorded content necessary to convey the "message"?

–   Is unnecessary repetition avoided?

–   Is a visual language also used with a mix of text, photos,graphs, tables?

–   Is there a "way" to get even more information?

Poster makers sometimes put a short text next to the poster or a QR code that can be used to download the poster and / or additional information (see additional explanation via https://www.qr-code-generator.com/qr-codes-on-posters/).

Authors often give hints and tips on the use of color, fonts, font size and aesthetic effect. In any case, it is important to choose a structure that suits your domain of knowledge.

We are nearing the end of the section on design guidelines, referring to the article by Hubenthal, O'Brien and Taber (2011). What is astonishing about their approach is that they presented theory-based design guidelines. In this case, Cognitive Theory of Multimedia Learning (CTML van Mayer, 2009). This theory emphasizes that everyone processes information simultaneously through different sensory channels: e.g. reading text and watching animations; reviewing visual elements and understanding the text. They stand out through design guidelines that fall into three dimensions:

–   Esthetical dimension

–   Cognitive dimension

–   The dimension of providing in time and space

You recognize the criteria for the aesthetic dimension, because they refer to the use of color, the use of images and the arrangement in which the flow (structure, coherence, order) is central. In addition to the cognitive dimension, the authors emphasize the connection with the learning objectives / curriculum, the extent to which the reader / viewer is examined or moved to think. To what extent are learning goals set so that a central idea emerges? Is there a balance between text and graphics that supports the content of the message? Are the graphic elements interconnected? In the dimension of rendering in time and space, the authors emphasize "continuation" after using and / or viewing / experiencing the posters. An example is the above QR code from which you can get additional information.

Useful tools for the creation of posters

It is always helpful to support students in designing adequate posters that match the design guidelines and criteria. This often results in the sharing of "formats (forms)" for the development of specific types of posters. Examples of such formats can be found for example, U.C. Davis University

(https://urc.ucdavis.edu/creating-effective-academic-posters). You can find more guidance, for example, at the following link at the University of Kent

(https://www.kent.ac.uk/brand/visual/posters.html). A variety of design guidelines can also be integrated into a checklist used to revise tasks related to this topic.

Power	Design Elements	Description	Reason
Attractive / Cognitive	Aesthetics	Visual appeal of the poster including; artistic design, color pallet, layout, overall size, print quality, etc.	- Encourages teachers to hang the poster - Contributes to overall classroom aesthetics - Attracts student attention
	Curricular Connection	Explicit and intentional connections to the curriculum including; broad overarching themes, process skills or individual content chunks	- Posit ions poster within curriculum and fosters a "sciency" classroom environment - Encourages teachers to hang the poster and rotate with topics/units - Affords both direct and indirect linkages to topics currently being studied
	Invitation to Inquiry	Uses cognitive learning theory to attract and engage learners in a minds-on way (e.g. title as question, visual analogy, or discrepant imagery)	- Activates students' attention and catalyzes cognitive processing; creates "need-to-know" - Provides a framework to guide students' construction of knowledge
	Message	Explicit learning objectives distilled to essential ideas	- Creates purpose - Defines what the learner should be able to do, know or believe - Distinguishes germane from extraneous text and visual content
	Text/Image ratio	Decrease volume of explanatory text and increase size of central imagery	- Reduces the visual complexity and lowers the initial cognitive load - Increases approachability (less investment) - Central or iconic images are visible to more students Complimentary visual, text, and pedagogical elements increase visual power and educational effectiveness - Fosters teacher/student/poster interactions
Protractive	Extensions	Provide opportunities for students to extend understanding through elements outside the 2-D, static space of a poster	- Integrates poster into instructional process - Offloads related but extraneous information from the poster to the web - Extends time on task - Fosters interactions with supporting live or simulated phenomenon

To a lesser extent, empirical research is available that systematically measures the effect of poster use on educational achievement.

Rosanti and Abu Seman (2019) set up experimental design in higher education and compared a group of students who studied learning goals with and without posters. In this small study, students who used a poster in the experiment performed better on a knowledge processing test.

A study by Banerjee and Greene (2013) investigated the effect of poster design in an anti-smoking campaign. An analysis of the content of the poster shows that adolescents have a fairly good idea of the impact of smoking and which anti-smoking campaigns are effective.

Riejos and colleagues (2001) required students to make posters in English in order to try to convey the metaphors in an understandable way. It turned out that the posters had a significant impact

on a better understanding of metaphors. Reilly (2007) also uses posters in classes of his language adapted to the target group — English as a foreign language. In his research, posters are mainly used as hints for words, sentences, ideas and situations. In addition, posters are used to promote interaction between students and teachers. Data analysis indicates that oral communication is stronger, that there is more authentic communication, that students are more confident, that there are fewer language mistakes, that students can work independently, that there is a good insight into students' oral language skills and the like.

Of course, the poster effect cannot be separated from the integrated use of posters in the wider learning and teaching environment. Nopr and Shahrill (2014) used the design and use of a poster presentation as a basis in mathematics (area: triangle geometry). They report a significantly greater learning effect on students on the test. However, it turned out that the results in the experimental conditions were significantly more positive than they were in reality.

Let's take a different approach now and watch a snapshot of the animation made using the Doodly app. With this you can develop animated drawings, add text, images and add voice. In the next topic, we will deal with the types of tools for the promotion of educational interaction and communication. In the topic, we immediately pay attention to the basic problem in joint learning: there is no "cooperation". The solution is immediately imposed: the development of a "script" that gives "structure" to group work and joint learning.

https://www.youtube.com/watch?v=kvSPorYXs9A&feature=emb_logo

−Discussion and collaboration in higher education

Discussion and collaboration in higher education facilitate the mastery of the following learning goals:

- Explain that providing a "structure" is essential for successful cooperation.
- Develop a script for content and / or a role that supports group collaboration.
- Draw attention to the fact that when working together, both individual and group responsibility are provided.

We will begin with a critical note of the oft-mentioned "group work." We do this by inviting you to read the article by Allan, EG (2016). One can often hear in students: "I hate group work!" The following article addresses students 'concerns about learning in small groups.

There are many blogs, websites, and research articles that similarly express the importance of group work. It is noticeable that not only teachers, but also students often express themselves negatively about group work (read https://eduadvisor.mi/articles/vhi-everi-college-student-hates-group-assignments/).

> Does shared learning have a positive effect on learning?
> What is needed for shared learning?

From both a theoretical and a research perspective, the potential for collaborative learning is a very high quality and even effective way of learning. In order to do this successfully, a number of conditions must be met.

A short introductory activity on a mind map brings us closer to research on collaborative learning. What does the research say? In his book, Hattie (2012) summarizes the results of 50,000 studies examining what "works" in education. His meta-analysis is clear: shared learning obviously has a greater learning effect than individual learning. But he immediately emphasizes that this really means that the critical preconditions have been met. From this and other research, you learn that it is necessary to have a clear goal in mind (what to do?). All participants individually and the group as a whole must be evaluated. Students have a need for structure when working in a group. They need feedback on their approach and group work. In addition to feedback on the final product to be delivered, the group needs to evaluate its own work. The wrong assumption is that students can work together. Collaboration is a difficult competence that you have to learn, so students need support in the process of cooperation. The key word that is repeatedly used in the literature to initiate joint learning is structure. In this thread we do this using scripts. We will talk about this in more detail in the next section.

Designing group excercises

In the first example scenario, we look at group work in the context of health training. In this type of training, students do a lot of practice and come to practical environments where they continuously deal with problems. In this example, students are continuously offered network problems over a long period of time, in addition to their internships, which are based on a well-chosen case. To prevent students from facing these problems too routinely and forget what was offered to them in their courses in terms of theory, research, procedures, ethics. They focus on group work. The teacher developed two types of scripts in this approach. In some cases, students work on the basis of content scripts. In other cases, they work according to specific roles commonly encountered in practice.

The teacher determines the specific roles for each student: the role of the pharmacist, the role of the teacher, the role of the researcher, the role of the representative of the pharmaceutical company. When dealing with a case, each student responds from their role. The "pharmacist" will have to follow the correct role when, for example, he checks the doctor's prescription. The role of the "teacher" means that the student continuously checks whether what others write is in line with what has been covered in class. Therefore, he / she supports proposals, approaches, definitions, procedures, based on specific documents from theoretical and / or practical lessons. The "researcher" seeks additional information on the Internet to encourage or support a group approach. The "sales representative" is looking for alternative products and / or tools that can be offered / sold in a specific situation. An alternative approach to roles requires students to take on the following roles: "moderator", "source researcher" (in courses and on the Internet), "examiner" (this student continuously asks other group members what they represent) and "integrator" all hitherto achieved in an ever-increasing summary).

Content script

The emphasis is on the tasks that are solved when solving problems: e.g. defining problems, presenting hypotheses, proposing a procedure for collecting / measuring, collecting data, summarizing the results, discussing and writing conclusions. In the upcoming example, there is a situation that students are dealing with. A seventy-year-old man, Mr. Suikerman, enters the pharmacy with a doctor's prescription. The man is a patient with diabetes. He will usually be given injections, but is now being given a prescription for an orally administered drug.

LKC
Level of Knowledge Construction

Research on this approach to group work shows that students working with roles integrate a much greater degree of knowledge and interaction in their further work. Second, research also shows that not everyone learns the same, depending on the role. In the case studied here, you see that the level of knowledge development is lowest in the student who takes on the role of moderator, while the student who takes on the role of "examiner" and "integrator" obviously learns much more. This is obvious, because the last two follow very carefully what all the other members of the group are doing. Based on this research result, advice is given for "rotating" roles between group members.

Here we have developed a series of example scripts. Depending on the task in this topic, it is not just a general description in the script, but a 'script map' should be created for the general script. The point is that students "learn" how to approach the problem. Giving a clear procedure to the task should help them do so.

When choosing the content of the script, you should consider the steps to be taken in solving the problem/task:

- Interview: determining the purpose of the interview, selecting the people to be interviewed, developing interview guidelines, taking notes during the interview, summarizing the answers per interview, developing conclusions for each interview.

- Photo collage for the analysis of a country's economy: defining economic sectors, defining the basic concepts and processes for which photographs are sought, searching for photographs, developing a common thread for a photo exhibition, determining the comment text for each photograph, developing the text of the exhibition brochure ...

- Video (5 minutes) for the play: reading a play, selecting key actions for each act of the play, writing a short script, division of roles in a group, scenography and costumes, shooting a video, editing a video, making initial and final credits. .

- Economic problem: market demand analysis, product market analysis, new product prototype development, prototype market research.

Role-defined scripts

Roles in the discussion help students to better understand their speech and to develop communication skills - speaking and listening.
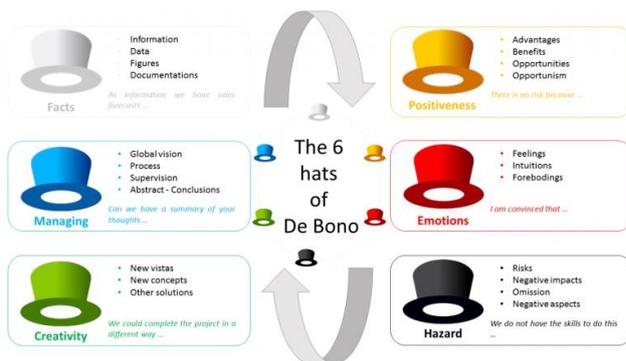
Examples of videos about "roles in group work" can also be found in Klascement (e.g. https://www.klascement.net/downloadbaar-lesmaterialen/94290/rollen-groepswerk-kaartjes/).

When using a script, it is important that the structure is clear to the students and that is why the script is written in great detail and step by step.

Researchers Johnson & Johnson (1996) and Slavin (1996) also point out the following characteristics of group work design. These researchers conducted a meta-analysis of research on the results of joint learning success. Their publications are still considered a milestone in the research of teaching methods. The following figure shows how researchers come to very similar conclusions.

This would be our selection of guidelines:

- Provide a very clear and operational (discernible) learning goal so that students know well what they need to provide in the end. Slavin emphasizes this as "group goals".

- Positive interdependence means that collaboration is beneficial and that it does not fail only in the distribution of works without interaction. This does not mean that students do everything together, but there must always be a phase in which individual work is combined into a puzzle, which leads to further improvement and that individual work is also evaluated on the basis of quality.

- "Individual" and "group responsibility" means that there is an explanation of teacher assessment that emphasizes what is assessed on the basis of individual work and what on the basis of actual cooperation.

- "Responsibility" also means that students are responsible for the quality of their work. Therefore, many teachers also offer a section that students use individually and / or in groups to evaluate their work before achieving the final result. If you want to see examples of column types, you can do so here: https://www.vernieuwenderwijs.nl/rubrics-klas-zo-ga-er-mee-aan-slag/.

- „Developing interpersonal skills "means that the instructions also help students to develop the competence to work together. For example, providing a script also leads to the development of this aspect of collaborative competence.

- „Task specialization "directly refers to the provision of a structure as described in the" script "explanation.

- „Introduce face-to-face contact ": A lot of group work fails because the teacher has no insight into the actual activities. When as a teacher you do not have regular contact, you do not get the opportunity to ask questions or get explanations and then no temporary feedback is provided. This often leads to "failure" or less successful group work.

Set a clear time. Nothing is more harmful than scheduling a meeting to do group work "within four weeks." We bet that most groups will postpone their work and therefore start too late.

References

1. Allan, E. G. (2016). " I Hate Group Work!": Addressing Students' Concerns about Small-Group Learning. InSight: A Journal of Scholarly Teaching, 11, 81-89.

2. Anderson, D. R., & Davidson, M. C. (2019). Receptive versus interactive video screens: A role for the brain's default mode network in learning from media. Computers in Human Behavior, 99, 168-180.

3. Blackstock, D., Edel-Malizia, S., Bittner, K., & Smithwick, E. (2017, June). Investigating interactive video assessment tools for online and blended learning. In International Conference on e-Learning (pp. 31-39). Academic Conferences International Limited.

4. Boucher, M., Creech, A., & Dubé, F. (2019). Video feedback and the self-evaluation of college-level guitarists during individual practice. Psychology of Music, 0305735619842374.

5. Eriksson, P. E., & Eriksson, Y. (2019). Live-action Communication Design: A Technical How-To Video Case Study. Technical Communication Quarterly, 28(1), 69-91.

6. Gordon Neil, Flexible Pedagogies: technology-enhanced learning, University of Hull, January 2014.

7. Haagsman, M. E., Scager, K., Boonstra, J., & Koster, M. C. (2020). Pop-up questions within educational videos: Effects on students' learning. Journal of Science Education and Technology, 1-12.

8. Hattie, J. (2009). Visible learning: a synthesis of over 800 metaanalyses relating to achievement. London: Routledge.

9. Hattie, J. (2015). The applicability of Visible Learning to higher education. Scholarship of teaching and Learning in Psychology, 1(1), 79.

10. Hattie, J., & Clarke, S. (2018). Visible Learning: Feedback. London: Routledge.

11. Hattie, J., & Timperley, H. (2007). The power of feedback. Review of educational research, 77(1), 81-112.

12. Mahoney, P., Macfarlane, S., & Ajjawi, R. (2019). A qualitative synthesis of video feedback in higher education. Teaching in Higher Education, 24(2), 157-179.

13. Masaki, M., Hechler, P., Gadbois, S., & Waddell, G. (2011). Piano performance assessment: video feedback and the Quality Assessment in Music Performance Inventory (QAMPI).

14. Moussiades, L., Kazanidis, I., & Iliopoulou, A. (2019). A framework for the development of educational video: An empirical approach. Innovations in Education and Teaching International, 56(2), 217-228.

15. Özkul, S., & Ortactepe, D. (2017). The use of video feedback in teaching process-approach EFL writing. TESOL Journal, 8(4), 862-877.

16. Pereira, J. A., Merí, A., Masdeu, C., Molina-Tomás, M. C., & Martínez-Carrió, A. (2020). Using videoclips to improve theoretical anatomy teaching. European Journal of Anatomy, 8(3), 143-146.

17. Preradovic, N. M., Lauc, T., & Panev, I. (2020). Investigating interactivity in instructional video tutorials for an undergraduate informatics course. Issues in Educational Research, 30(1), 203.

18. Schellens, T., Van Keer, H., & Valcke, M. (2005). The impact of role assignment on knowledge construction in asynchronous discussion groups: A multilevel analysis. Small Group Research, 36(6), 704-745.

19. Tang, T., Tang, J., Hong, J., Yu, L., Ren, P., & Wu, Y. (2020). Design guidelines for augmenting short-form videos using animated data visualizations. Journal of Visualization, 1-14.

20. Ten Hove, P., & van der Meij, H. (2015). Like it or not. What characterizes YouTube's more popular instructional videos?. Technical communication, 62(1), 48-62.

21. Timmers, S., Valcke, M., De Mil, K., & Baeyens, W. R. G. (2008). The impact of computer supported collaborative learning on internship outcomes of pharmacy students. Interactive Learning Environments, 16(2), 131-141.

22. Valcke, M. (2018). Onderwijskunde als ontwerpwetenschap. Deel I. Leuven/Gent: Acco.

23. Weinberger, A., Ertl, B., Fischer, F., & Mandl, H. (2005). Epistemic and social scripts in computer–supported collaborative learning. Instructional Science, 33(1), 1-30.

24. Zainuddin Z., Halili S. H.(2016). Flipped Classroom Research and Trends from Different Fields of Study, International Review of Research in Open and Distributed Learning Volume 17

25. The power of blended learning, July 2020, Professional innovation pathways, e-Workbook

Authors:
Slađana Dimitrijević
Ana Kaplarević-Mališić

QUESTIONS:

How to increase students' active participation in classes?

How to additionally motivate students to learn?

How to enable students to learn autonomously?

How to impact students to take responsibility for their own work and progress?

Can the answer be found in the blended model of teaching and learning?

What is *Blended learning* and how to organize it?

What is the *Flipped classroom* and how to organize it?

Blended learning
with special reference to the *Flipped classroom*

Contents

1 Preface

It can indeed be said that the educational theory and practice have been constantly transforming since the establishment of the first school until today, and that it is evident that this transformation has been increasingly gaining momentum in the last several decades, becoming more demanding in the process. This has primarily been caused by the changes in the life conditions of entire societies, in fact the whole humanity, setting new educational goals and outcomes. Furthermore, dramatic circumstances connected with the COVID 19 pandemic have shown how important the flexibility of one of the vital systems in any society is, the system which determines the very fate of the society's future – the educational system. All this has additionally motivated, but also forced, teachers around the world to integrate technology in their instruction regardless of how willing they were to take such a turn, or how prepared they were to implement new strategies.

This short handbook was written as a segment of the work in the TeComp ERASMUS + project, which was dedicated to the enhancement of teaching competencies of individuals working in the higher education, and one of the motives for selecting *Blended learning* and *Flipped classroom* as the topics of this handbook can also be found in the unexpected circumstances which we faced during the realization of the project. What had been imagined as a process intended to last for several years – implementation of online teaching elements into the regular work, suddenly became our everyday life. We therefore believe that this experience with online teaching has enabled us to more adeptly design and prepare classes which integrate modern technologies in regular circumstances. The aim of this material is to provide a brief and informative overview of *Blended learning*, with special reference to one of its realizations – the *Flipped classroom.* The recommended models represent an illustration of a theoretical framework and are conceived as a foundation for the work and discussion in workshops which will be realized as a part of the project. It is expected that this discussion will result in concrete examples adapted to the needs of the participants in the workshops, and also to their students. We hope that the material will be useful as the aid for class planning, and we would be especially pleased if the reader is driven to critically consider the use of various teaching strategies and models and if this results in professional or scientific papers on the subject.

Introduction

In its broadest sense, the education as a whole is a directed and organized pursuit aimed at enabling the students to gain certain competencies, i.e. to enable the students for an independent life in which they will be able to realize their potentials and be valuable members of the community to which they belong. Any type of education has three aspects: the students acquire knowledge, develop faculties and form attitudes, i.e. they gain competencies. Although education represents a civilizational constant, its form and the competencies that stem from it have changed over the course of time, which in turn depended on the changes in life conditions, both natural and social.

One of the dominant features of the modern age is the omnipresent influx of technology into our lives (both on the individual level and the level of the entire society), as well as the exponential increase of the available information. This also affects the change in instruction and teaching methods, so that the education results in what could in brief be called the key competences of a 21st-century individual. Here, we see competences as multifunctional combinations of knowledge, skills and attitudes which individuals require for their personal affirmation and development. The following key competences can be distinguished:

- communication in the mother tongue,
- communication in a foreign language,
- mathematical literacy,
- literacy in science and technology,
- problem solving,
- IT literacy,
- "learning to learn",
- health literacy,
- "entrepreneurship",
- social competence (cooperation and team work),
- civic competence,
- cultural awareness, aesthetic competence,
- ecological competence.

It can be observed that numerous listed competences are not exclusively connected to a single educational subject, and that they need to be developed in a group of subjects that is as broad as possible. Some of these are for instance problem solving competence, "learning to learn" competence and social competence. A question arises about how to accomplish this. One of the answers which the contemporary pedagogic and didactic theory states is the use of teaching strategies which will support active learning, which will put the student into a position of the key participant in the process and the key agent in activities, while the teacher is more likely to play the role of the organizer and coordinator of the learning process. At the same time, it is recommended that the instruction suits the current moment, the environment in which students live today and their habits, so it is recommended that there is a purposeful and adequate introduction of the technology into education. Actually, what is intended is to find a way to maximize the use of the good sides of the face-to-face traditional teaching combined

with the new potentials of providing information and cooperation by using modern technologies, or more precisely to form a synergy between traditional and online classes. Briefly put, it is an idea from which a new model of education has emerged, a new strategy of teaching and learning, the so-called *blended learning.*

2   What is blended learning?

It is important to note that the origin of *blended learning* predates the advent of digital technology despite being closely associated with it nowadays. The genealogy of *blended learning* can be found in the distance learning through correspondence courses. For example, the children of lighthouse keepers in Canada were educated in this way as early as 1919 (Barbour, 2014). The objective of overcoming the spatial distance still remains one of the major motives for the use of blended learning. An additional incentive for the development of such a type of teaching happened at the end of the previous century, when the availability of personal computers and the emergence of the Internet and social networks made the development of new teaching models and learning at the various educational levels possible. The new technology had the potential not only to cross the space, but also to cross the time (by recording) and to individualize learning (students have the control to choose their path through the curriculum and to choose their learning pace). All this has had an impact on the teachers belonging to different areas and different pedagogic philosophies to implement new technologies in their teaching practice for various reasons, and therefore develop different notions about what *blended learning* actually involves.

It is rather obvious what the origin of the label *blended learning* is. Essentially, this approach involves the blending (combining) of:

- face-to-face learning activities with online activities and formats,
- traditional teaching schedule with some other types, such as working at weekends, intensive work, supplementary work,
- conventional technologies, such as note-taking, combined with social networks and modern technologies,
- simulations, group activities, learning by using sites, practical exercises.

Most broadly understood, and most probably the most profound and useful interpretation of the *blended learning* is the one where it needs to include a combination of various teaching theories in educational practice in a way that most adequately suits the given situation. What needs to be kept in mind is that learning theories are not like a religion, one does not exclude the other, and the aim is have a proper theory for a proper situation (Zemke, 2002). The choice in a specific situation depends on the characteristics of the students, educational field and concrete teaching content, as well as the nature of the knowledge and skills which the students need to acquire and the context in which these are meant to be applied.
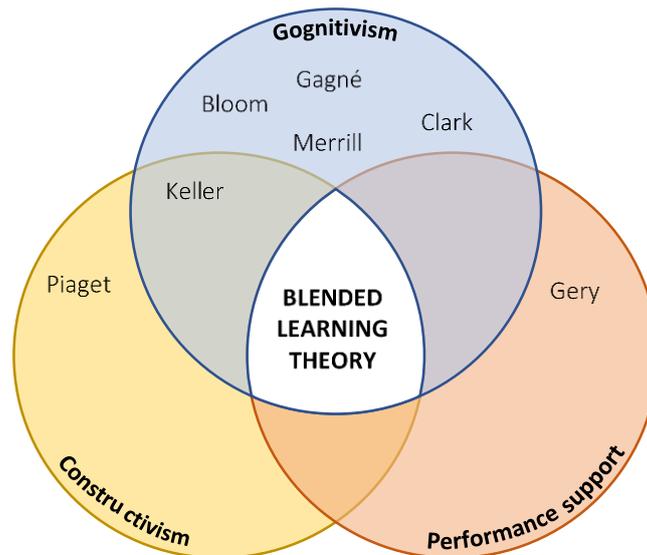
Picture 1. Blended learning as a combination of multiple learning theories

Due to all mentioned above, it is evident that there is no generally accepted and complete definition of the term *blended learning*. Today this term is predominantly used to signify the integration of face-to-face learning (in a real classroom) with online or e-learning.

Blended learning is a teaching model that relies on the strategies and systematic approach of combining time and types of learning, so that the best aspects of traditional face-to-face teaching, i.e. the teaching in a real classroom, and the online interactive teaching, using suitable digital and communication technologies, are combined.

It is estimated that the future learning systems will be differentiated not by taking into account whether they combine different strategies, but how they combine them. The question how this is done is by all means one of the most important ones. Like any problem with the design, this challenge depends on the context for the most part and has a virtually infinite number of potential solutions.

New technologies (ICT) can be powerful pedagogic tools, not only as valuable sources of information, but also as the enhancement of human abilities and contexts for social interaction which support learning. However, the process of using technology for the enhancement of learning never is and never should be merely a technical issue. What certainly needs to be avoided is the introduction of new technologies without a clear didactic justification and goal. A mere introduction of ICT does not necessarily mean that the improvement in the teaching process will occur. Many authors have already undertaken empirical research to measure the effects of introducing technology in the education and the results are not uniform. Just like other types of innovations, they bring success only when they are theoretically justified and well designed, since otherwise there is no moving forward and also negative effects can occur.

−Key elements of blended learning

Five key factors can be distinguished as the important elements of blended learning:

1) **live events**, which are synchronous learning events headed by the teacher, where the students participate concurrently in a real or virtual classroom;

2) **self-paced learning**, i.e. learning where the student is autonomous, learning at his or her own pace at the time which is the most suitable for him or her;

3) **cooperation**, students communicate among themselves in an appropriate format and environment, e.g. by email, as a part of a forum or on social networks;

4) **grading/assessment**, the evaluation of students' knowledge, which can precede the live events and self-paced learning, so in these instances they aim at reinforcing the previous knowledge on which the new knowledge builds, while subsequent instances of testing the knowledge are obligatory in order to determine the accomplished level of knowledge and measure the achieved knowledge transfer;

5) **performance support materials**, which can be diverse: printed, electronic, linear, non-linear, interactive or not.

Picture 2. Key elements of the blended learning

Synchronous live events represent the central factor of blended learning. For many students nothing can replace live lectures held by a competent teacher, i.e. nothing else can be as stimulating for their learning. How can one accomplish an efficient live event? There is no simple and unique answer to this question. John Keller, a theoretician (Carman, 2005), provides an answer through his ARCS motivation model consisting of four elements: Attention, Relevance, Confidence, Satisfaction. Each element of this model can be used to create an engaging and efficient live learning experience.

1) *Attention*. The first aspect of the ARCS model is gaining and maintaining the students' attention. For example, an experienced teacher in a real or virtual classroom can begin his or her class with an anecdote, an interesting historical event related to the content that is presented or by asking questions which will inspire students to think about the desired topic – a synchronous discussion in a real classroom or online to prepare them for learning.

2) *Relevance*. Students remain focused when they believe that the content that they learn is relevant for their specific situation. In order to show relevance, the teacher can use examples or analogies which are familiar to the students. The students need to be instructed, or better yet demonstrated how they can use the information from the course to solve real problems in practical situations.

3) *Confidence*. Students must be confident in their skills and abilities in order to maintain motivation and the goal/outcome must be obtainable. The teacher needs to clarify what is expected of students, what the intended outcomes are, and then to leave sufficient time for them to achieve this, so that they can learn what was planned and gain good command of their new skills. Like George Pólya, a renowned mathematician and mathematics methodologist, used to say: a student will not start loving mathematics unless he or she experiences the joy of "victory" over the given problem. Self-confidence is gained through experienced success.

4) *Satisfaction*. In order to keep the students motivated, it is important for them to be satisfied with the entire learning experience. One of the ways to achieve this is to provide different oportunities to learn, but also to provide conditions for a successful application of the learned content.

Asynchronous autonomous learning events lend a substantial additional value to the blended learning model. In order to achieve the maximum effects of autonomous learning, it needs to be properly prepared and organized. Teaching content cannot be simply divided into synchronous and asynchronous activities and have expections that such teaching will be purposeful. The modern theory of curriculum design supports the use of multimedia as a tool which promotes the transfer of knowledge. In this respect, three principles deserve a special attention (Ruth Clark, 2002).

1) Multimedia principle – adding: a graph in the text can improve learning. Research has shown that graphical images can enhance learning. Here, it is essential to ensure that the graphical images are directly related to the information from the text, to convey a specific message or instruction, more precisely to educate, not to embellish.

2) Adjacency principle – placing: the text immediately next to the graphical images enhances learning. Numerous studies (Clark, 2002) have confirmed that the learning from materials which have integrated words in the adjacency of visual elements has resulted in a substantial improvement in learning.

3) Modality principle – explaining: a graphical image with an auditory explanation improves the learning. Auditory explanations should especially be used in situations when it is likely that the students will have problems acquiring the teaching content, either due to a large amount of information or the complexity of the content. For instance, if students watch a video, for example a software demonstration in several steps, the students' attention is focused on the animation, so they will be overwhelmed by having to read off the screen, so the auditory explanation is preferable in such cases. Similarly, providing a long or complex proof of a theorem is more successful if it is accompanied by an auditory comment, to emphasize the main proving idea and process behind it, as well as the most important steps.

Purposeful cooperation intensifies the force of a live event or an independent learning experience. As the constructionist theory of learning explains, people, as social beings, develop new notions and knowledge thorough the social interactions with the community, i.e. other people. Joint (collaborative and cooperative) learning provides additional possibilities which are not present in traditional instruction, since the group can achieve purposeful learning and solve problems better than any individual alone (Tinzmann & al.1990). When organizing the blended learning, the teacher needs to create an environment in which both he/she and the students can cooperate synchronously (groups in a real classroom, chart rooms in a virtual one, and the like) and asynchronously (email, forum panels, and the like). Two types of cooperation can be distinguished: student–student and student–teacher. Cooperation on the student–student relation enables the students to converse among themselves about critical issues, and in doing so sometimes learn the part of the content. Cooperation on the student–teacher level provides the teachers with the opportunity to teach personally, to individualize their classes according to the needs of every respective student, while the students gain an option to get additional personalized learning support, which can also include additional guidelines in the form of advice, reminders or suggested exercises.

Grading/assessment is one of the most crucial factors of blended learning for two reasons:

1) it enables students to "try out" the content which they already know, and thus adjust blended learning to their own needs, and

2) it measures the efficiency of all other modes of learning.

In 1956 Bloom provided one of the most widely used frameworks for designing and giving assessments, which later underwent several modifications (see Picture 3), but it is still considered to be a generally accepted system applied as the cornerstone when formulating the learning outcomes or assessing the achievements of students.
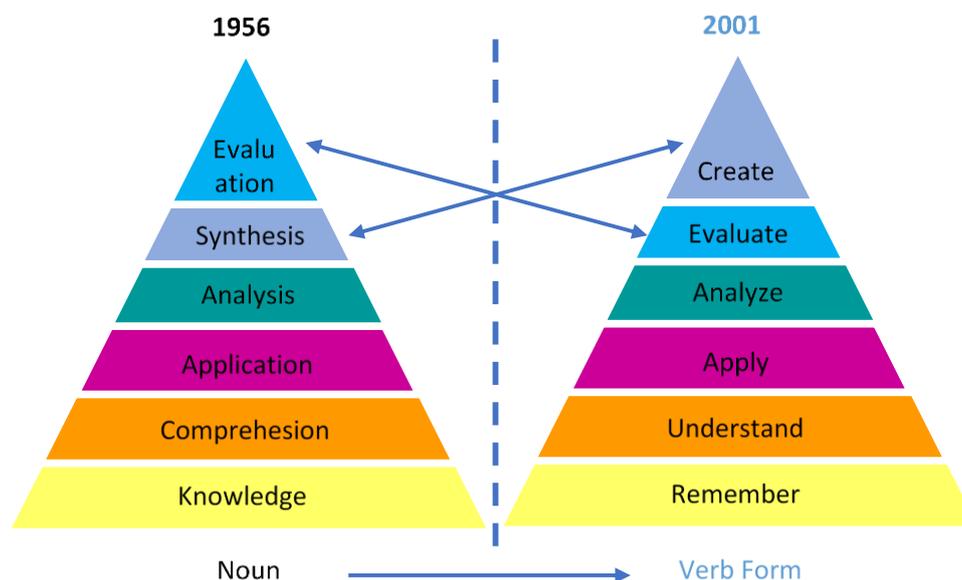
Picture 3. – A modification of Bloom's taxonomy

Six levels of cognitive learning that are most frequently identified are: knowledge, comprehension, application, analysis, synthesis and evaluation (see Table 1).

Level	Description	Activities of students
1. Knowledge	Recalling and knowing content that has been previously learnt.	Recognize, name, state, show, cite, repeat, find, list, choose, learn by rote
2. Comprehension	Understanding the content, usually demonstrated by explaining what has been learnt by using one's own words and illustrating by using one's own examples.	Explain, distinguish, organize, demonstrate, conclude, predict, connect
3. Application	Applying what is learnt (rules, principles, formulae, theories, concepts, procedures) in a new context, in order to solve a given problem or to respond to questions or some other request.	Apply, solve, adjust, do in another way, expand, use, exemplify, provide new examples, group
4. Analysis	Breaking a whole into smaller parts accompanied by an explanation of the connections between the parts.	Make a table/a graph/a structural model, differentiate, classify, compare and select
5. Synthesis	Assembling the parts into a new whole by creating a new structure.	Create, construct, explain and present, summarize, plan, modify, arrange, combine
6. Evaluation	Applying various criteria in order to assess ideas and to propose substantiated conclusions and attitudes.	Assess, decide, opine, measure, rank, suggest, conclude

Table 1. Bloom's taxonomy – the classification of learning levels

Performance support materials are probably the most important element of blended learning. The following can be distinguished: printed materials or materials for print, supplementary materials and resources, and multimedia materials. Printed materials are an indispensable choice and a part of every learning, but for some time now electronic materials prepared for print, but not necessarily printed, have been used. Supplementary materials can be synopses, check lists that are used to monitor the progress or graphs in which the obtained results can be registered, etc., and they are aimed at aiding the learning process. By the introduction of technology into the education, additional space for the multimedia teaching resources was created, as they have made, with the increasing availability and popularity of personal digital assistants (android mobile phones, tablets, laptops), possible for every student to have a prompt support for their learning at any moment and place.

–Blended learning design

Regardless of the applied model, teaching always represents an organized process and without a good preparation,favourable outcomes are highly unlikely. What should especially be taken into account in the process of lesson planning by applying blended learning is the following:

1) **The role of physical space**. Use the face-to-face interaction when the activities are most suited to the use of physical space, classrooms, laboratories, computer rooms. Bear in mind that some of these spaces represent communal educational resources where students can work and learn together (**collaborative and cooperative learning).**

2) **Planning** and designing teaching materials. Along with the material that you have created yourself, also consider open educational resources and resources available to students in various libraries.

3) **Use of online spaces**. Use spaces and services for cooperation and communication, available at respective college or university level, as well as general open services. Discussion forums should be formed, as well as information exchange blogs, joint/shared notebooks, wiki pages, etc.

4) **Providing feedback**. Feedback must be timely, clear and concise.

5) **Flexibility vs. structure**. Often when one adds flexibility, there may be a certain loss of structure. Therefore, setting deadlines, clear learning goals and expectations from students play an important role in blended learning.

6) **Class participation vs. class attendance**. Students must understand what is expected from them and what the educational requirements for the successful completion of a class are. They need to be clearly told that participation is important and required.

7) **Working hours**. Working hours are not restricted solely to classes, students need to be given additional help or opportunity to ask questions. For this purpose, consider using asynchronous and/or synchronous sessions.

Blended learning combines traditional face-to-face instruction with online learning, so that the students can benefit from the teacher's knowledge and guidelines, but also be able to individually organize learning which involves the use of various educational platforms or applications that assist with learning. However, there is no unique combination of all these elements, as blended learning depends on numerous factors (teaching content, learning context, educational resources, students' previous knowledge and many other). Actually, it can be said that blended learning is a concept, an

umbrella term, comprising several other submodels. Some typical instances of blended learning will be given and briefly explained below.

1) **Flipped classroom**. Flipped (inverted, upside down) classroom is a methodology, a learning approach in which the technology is used to flip (change) the traditional role of a classroom. If in the past the time spent in the classroom was dedicated to lecturing the students, in the inverted model the classroom is now used to stimulate individualized learning and to provide individual support to students, as well as to enhance interaction among students, and between students and teachers. Although the teaching content still remains a class segment, this content is mostly designed in such a way to be available outside the classroom as well, beyond classes, which is an excellent way for students to learn at their own pace.

2) **Rotation model**. In this model the students rotate taking part in various activities in fixed time intervals, where at least one of these activities involve online learning, while the others may consist of work in small groups or frontal work, group projects, individual instruction, doing homework and exercises. In some instances, the entire group alternately switches activities, while in others there are rotations of small groups of students. In any case all students undergo all activities. A rotation may include the rotation of work in a classic and virtual classroom on a fixed schedule.

3) **Flex model**. In this model, online learning is the basis of learning, but the student is occasionally directed to the activities in a real classroom, and the schedule is not precisely determined. They are able to flexibly move through various modalities of learning with the aim of optimizing their learning experience based on their specific needs. Each student basically has an adapted, fluid schedule in respect to learning modalities. The teacher provides face-to-face support to students on a flexible and adaptable basis through activities such as instruction in small groups, group projects and individual instruction. In some cases, there is a considerable face-to-face support, while in others it is minimal.

4) **Project based learning**. In the foundation of this model is the work on a concrete project and the learning is organized according to the needs that the realization of the project requires. A student creates a query, and then uses online resources to gather information. The teacher is available to students for support in all stages of the project, especially when designing the activity plan. This is one of the ways for the lectures to be more intensively connected with the real life and to demonstrate the application of theory on real problems.

Like any other approach to instruction and learning, blended learning also has its benefits and drawbacks. What needs to be borne in mind is that the combination of autonomous and face-to-face learning can be useful, but only when applied adequately. There are risks, since not all students are ready to act independently when learning is concerned. For some, this type of learning has a motivational impact, while others feel confused. It is evident that blended learning can be more available and flexible for students, that it can increase the level of active learning in class, and when the teacher is concerned, blended learning can enhance the instruction and class organization. We believe that the advantages of this model surpass the drawbacks, and that in each specific case the disadvantages can be minimized. The key for success is that teachers pay close attention to their students and provide them with strong support.

The greatest advantages of using blended learning are the following:
1) **Individualized support for students.** As we mentioned before, the strongest point of blended learning is the opportunity to provide each student with the personalized instruction which suits his or her needs.

2) **Students have access to useful electronic materials anytime and anywhere.** Modern personal technological devices enable the students to be available for learning at almost any time and place.

3) **Students can enjoy all the benefits of distance learning, i.e. the use of digital materials.** This can include the use of various sources of knowledge, from the available scientific libraries to essay data bases, at any time they need them.

4) **Blended learning involves more interaction and cooperation among students during learning.** Students have an opportunity to communicate both with the teachers and among themselves in a real or virtual classroom (using video conferencing or other means of communication). This contributes to a better cooperation between teachers and students, as well as among the students themselves.

5) **Students have much more time for learning at their disposal.** In blended learning the learning process is organized in a longer time span and extends beyond the end of the class.

6) **Some teachers claim that this method showed them how to be better instructors.** Some of the university teachers claim that the application of blended learning has motivated them to create better lessons, to think more about the experiences of the students, to make individual plans for students and help them learn.

The greatest disadvantageous factors of using blended learning are the following:

1) **Technology can pose a greater challenge than it is helpful.** One of the major problems can be technological literacy, which can significantly vary form one person to another even in this day and age. It can equally represent a problem for teachers and students, since not all of the digital sources are reliable nor easy to use. Furthermore, digital resources (devices, software, and the Internet) available to an individual can also vary greatly.

2) **Blended learning imposes additional obligations to the teachers.** There is much additional work for teachers in all stages of blended learning. Teachers must expand their perspectives, choose the most suitable curriculum and invest considerably more time and effort to find the right balance between the online learning and classic face-to-face classes. A great deal of motivation is also required for someone to move away from his or her adopted teaching style.

3) **Students can also be more burdened.** With a wide range of opportunities that a blended learning model has to offer, it also usually includes an increased number of different educational activities for students.

4) **The credibility of sources of knowledge and plagiarism have become an increasing problem.** Participating in a digital educational environment may lead to more plagiarism than usual, from various online sources. Actually, there is a whole range of unreliable sources of information which offer false or misinterpreted claims.

–Designing university courses which include blended learning

Investigating one's teaching practice should be an integral part of every instructor's work. The evaluation is not only directed to students' knowledge, skills and attitudes, but also to the entire teaching process, all of its participants, and it includes a reflexive consideration by teachers with regard to their own work, by analyzing what has been planned, realized and achieved. Within every scientific discipline there is a branch scientifically and exploratory dedicated to the teaching of the discipline in question. Unfortunately, the number of articles related to the university level of education in journals dedicated to teaching and education is the smallest. What dominates is the research concerning the

teaching practices in primary and secondary schools and as a rule there are more papers dealing with the teaching of the younger, or better still the youngest, pupils. Although many conclusions from these papers can almost fully be applied to the higher education as well, higher education has some specificities and it certainly deserves a special attention of the researchers. When we talk about the investigations that concern blended learning, the situation conforms to the description given above, thus leaving a large space for the potentials of this model to be further researched.

For this occasion, we will provide the conclusions of the paper by Alammary & al. (2014), analyzing different designs of blended learning applied to the higher education courses. What is also emphasised here is that there is no generally accepted definition of the model, so the teachers in the higher education developed different interpretations of this concept, after which different designs were developed. What represents the main challenge for numerous teachers who have only started to adopt the idea of blended learning is the appropriate choice of the course designing approach.

The teachers' understanding of the blended learning notion usually represents the foundation for the course design that the model is intended to promote. Therefore, for example, teachers who see blended learning as a simple method of combining the traditional learning with online learning, add additional online activities to their traditional face-to-face courses and call this blended learning.

> Investigating different processes of designing combined learning courses, three different designing approaches can be distinguished:
> 1) **low-impact of blended learning** – adding supplementary activities to the existing course,
> 2) **medium-impact of blended learning** – substituting activities in the existing course,
> 3) **high-impact of blended learning** – building the course from scratch based on the strategy which relies on the blended learning model.

The classification of these approaches has been made in accordance with the potential changes in the existing curriculums and students' learning experiences. In the following lines, all three approaches to the course design have been briefly considered, several benefits and challenges have been identified, which in turn give us an opportunity to formulate recommendations for teachers.

Low-impact blended learning approach – adding activities

In a low-impact approach, additional online activities are added to the traditional face-to-face course. The study conducted by Kaleta & al. (2007) revealed that most of the instructors who design "blended" courses add online components to their traditional courses without eliminating any of the existing activities. They called this phenomenon "course-and-a-half syndrome". It was observed that adding online activities to the already established course usually happens when inexperienced teachers form their first combined learning course. Simply by adding, these teachers try to benefit from the advantages of blended learning without investing effort to re-evaluate overall aims of the course in the context of blended learning model. However, there were cases where the added activity happened to be a result of a pedagogical need and turned out to be a valuable addition to the traditional course.

Benefits

i. It is an easy approach to designing combined learning courses, which potentially encourages teachers who still hesitate (because they think that blended learning is quite complex and highly technical) to include blended learning.

ii.   It is a quick approach to producing a blended learning course. Led by a specific pedagogical need, teachers can directly add a new activity which satisfies that need in a suitable manner, without spending additional time and effort in rethinking and re-planning the entire course or in exploring numerous possible combinations of the blended learning components and methods of delivery.

iii.   There is a low risk of failure when it is applied carefully. According to Vaughan (2007), there are three major risk factors which were identified by teachers who had taught "blended" courses: the fear that students will get lower marks, the fear of losing control over the course, and uncertainty concerning the way the online learning would affect relationships in the classroom. By adding an activity while at the same time keeping the traditional course almost the same can minimize such risks.

iv.   Minimal experience in teaching the traditional course is sufficient to design a "blended" course. Even with a quite limited experience the teacher can spot the part of the course which could be enhanced by an extra online activity.

Challenges

i.   Teachers are required to have some technological knowledge to successfully apply this approach. In order to successfully integrate technology into their teaching experience, teachers need the knowledge that can allow them to: determine which technological tool they need so that they could achieve a certain pedagogical goal, to know how to use the tool to help the student, to improve the students' ability to use adequate technological tools in different stages of the learning process (exploration, analysis and production), to choose and adopt technological tools that will allow them to recognize the needs and solve problems in relation to their own professional development.

ii.   There is a high risk of developing a two separate paths in the course. Adding the work on the network, the online work, to the traditional course without reducing time spent in class often leads to two separate courses, the online one and the face-to-face one.

iii.   Students can perceive adding an extra activity as a burden rather than a bonus.

iv.   Adding a new activity without removing the existing one can increase the instructor's workload excessively. Teachers can face time limits and great workloads that result from adding additional online teaching resources.

v.   Current official norms concerning teaching workload often do not recognize additional activities for the existing course, so consequently teachers do not receive compensation for their additional efforts (Amiel, 2007). According to (Lee, Lee 2008), inadequate compensation and lack of true incentives constitute one of the main factors negatively affecting the use of e-learning.

Recommendations

i.   The teacher should start the process of introducing blended learning into his or her courses by adding simple online activities, which can easily be managed both by them and their students, for example a discussion forum. Later, if more online activities are needed, a medium-impact approach that involves the replacement of activities could be considered.

ii.   The addition of an activity must be driven by a specific pedagogical need, and not, for instance, by technology for technology's sake. Teachers therefore need to find out what is problematic in their courses or what is missing and understand how to apply the technologies and teaching approaches to solve the problems.

iii. The added activity needs to be well integrated into the course. It is essential to consider the connection between what is happening in class and what is happening online (Kaleta et al., 2007).

iv. The course should not be overloaded with tasks and activities. It is worth the effort to explore students' opinions about the components of the course. Evidently, consistent and transparent communication with students about their opinions and expectations is essential for the success of the combined learning experience.

v. A low-impact blend is recommended for teachers who lack experience in the blended learning design. It is easy to implement and has a low risk of failure.

Example. The chart below illustrates a possible transformation of (a part of) a traditional course by applying the low-impact blended learning approach.

Traditional (face to face) course		adding	Low-impact blended course	
1	lecture (face to face)		1	lecture (face to face)
2	lecture (face to face)		2	lecture (face to face)
3	lecture (face to face)		3	lecture (face to face)
4	tutorial (face to face)		4	tutorial (face to face)
5	tutorial (face to face)		5	tutorial (face to face)
6	test (face to face)		6	test (face to face)
			7	Group discussion on Facebook or some other social network or available platform

---

Medium-impact blended learning approach – replacing activities

In the medium-impact approach, the existing course is redesigned by replacing some of the face-to-face activities with online components. The assumption behind this approach is that some parts of the course would be more effective as online activities. In certain cases, the face-to-face sessions that remain are conducted in exactly the same way, while in other cases there are certain changes in these activities.

Benefits

i. This approach enables teachers to start in a simple way and apply the blended learning strategy gradually, replacing components of course as needed.

ii. The experience which is gained by applying this approach can help teachers to become more confident in introducing a larger-scale blended learning applicable to the entire course (Ertmer, Ottenbreit-Leftwich, 2010).

iii. This is a useful approach for teachers with certain experience in blended learning design, who do not want to risk making considerable changes to their courses. Kaleta & al. (2005) have observed that teachers usually like to teach in the same traditional way which is familiar to them and find it difficult and challenging to allot a considerable amount of time and effort to develop a new course.

iv. It enables teachers to experiment with different approaches to learning and to use more types of educational technologies without losing all the advantages of the traditional course. Teaching and learning by employing technology in an appropriate and effective way represents a challenge, but can be improved with experience.

Challenges

i. Teachers need to have a good command of technological knowledge and certain amount of confidence in order to apply this approach, since it is not possible to go back to the previous teaching method. Ertmer, Ottenbreit-Leftwich (2010) have pointed out that the knowledge of technology is essential to make students' learning easier, although it is not necessarily sufficient.

ii. To replace and integrate the new course components requires considerable time and effort.

iii. There are no defined standards that would guide the decisions about how much or what part of the course can or should be replaced. Such decisions are influenced by many factors, mainly by the nature of the course content and the teacher's intentions (Vaughan, 2007).

iv. It is necessary for the teacher to have previous experience in teaching traditional courses. The key challenge in designing a blended learning course by using this approach is the identification of the course part that was one which did not work properly in the traditional format and then to reach the decision whether it could work better online. Having little or no previous experience in teaching the course makes this process difficult.

v. Intensive planning in the long run, as well as the observation and assessment of the course, are necessary for a successful implementation. Achieving a good balance between online components and face-to-face components is the result of a gradual process of introducing new resources or techniques to replace the existing ones and then assessing if the use of these new resources or techniques helps students in achieving the learning goals (Duhaney, 2004).

Recommendations

i. Teachers should start applying the replacement of activities gradually. They should start by moving a small portion of their syllabus to the online mode, which in turn leads to the reduction of the time (or amount of) face-to-face lectures, and then if needed perform the process of scaling until a harmonious balance between face-to-face and online teaching has been.

ii. The harmonious balance will differ from one course to another. Variations occur due to a number of factors, which include characteristics of the students, the teacher's experience, the teaching style, course goals and available online resources. In some courses there will be more teaching in the real classroom than the online one, while in others the focus will be on online components, and in some other cases there will be a more-or-less equally distributed mix of the two models of instruction and learning.

iii. Achieving a harmonious balance requires expertise and constant analysis, accompanied by the regular evaluation of the course. Iterative redesign of the course involves getting feedback as a crucial factor for the course enhancement. Teachers' experience accumulated in blended learning design can play a vital role in reaching this balance.

iv. This approach can be applied by teachers with medium to extensive previous experience in teaching the traditional course. During the process of replacement, numerous critical decisions need to be made, such as what portion or which part of the courses can be replaced (Vaughan, 2007), so the teacher's experience is essential when making the right decisions.

v. Institutional support is important for this approach to be successful. Providing technical support through the system is important, taking into account the teachers' workload and the fact that they have to face their own potential fears and resistance towards blended learning. Furthermore, the teachers' motivation for professional development is essential, so that they

can adopt new teaching methods and master new technological tools, and be competent to decide on the most appropriate methods of instruction delivery for the given course.

Example. The chart below illustrates a possible transformation of (a part of) a traditional course by applying the medium-impact blended learning approach.

Traditional (face to face) course			Medium-impact blended course	
1	lecture (face to face)		1	lecture (face to face)
2	lecture (face to face)		2	google – hangout or something similar
3	lecture (face to face)	replacement ➡	3	lecture (face to face)
4	tutorial (face to face)		4	Group discussion on Facebook or some other social network or available platform
5	tutorial (face to face)		5	tutorial (face to face)
6	test (face to face)		6	online test

High-impact blended learning approach – starting from scratch

In the high-impact approach the blended learning course is built from scratch. This approach has been described in the literature as a full redesign, total redesign or radical changes. A usual way of applying this approach was described by Harriman (2004) and Hofmann (2006). They recommended that instead of considering the entire course as a whole, the teacher should consider each respective course learning outcome. For each outcome, the teacher needs to determine the best way to achieve that outcome. They recommended that by applying this approach at the level of learning outcomes, teachers could get the most effective combination of technologies and could create a better curriculum. This approach conforms to the conventional curriculum development model, which is called constructive alignment, in which assessment tasks are aligned with the learning outcomes. Hofmann (2006) also added that it was wrong to assume that redesigning an existing course would take less time than building a new course.

Benefits

i. It provides an opportunity for improvement and reduction or elimination of problems which the existing course has. Teachers make a start from a fresh perspective and with better chances to design a more successful course, especially in the cases when the problems in the traditional course have been observed/recognized.

ii. It enables a better integration of online components with face-to-face components. According to Littlejohn, Pegler (2007), it is necessary to build the course from scratch in order to create the effective integration of face-to-face and online components.

iii. It provides teachers with the opportunity to get maximum advantages from blended learning and to better meet the needs of their students. Forming the course from scratch gives a better chance to reevaluate and redesign the entire course bearing in mind learners' needs.

Challenges

i. For a successful application of this approach, a high level of technological knowledge and confidence is required. With a high level of technological competence, teachers can easily master new technological tools and use them in their courses. Wozney & al. (2006) have discovered that one of the most important factors which influence the integration of

technology into teaching is teachers' belief that this technology can help them to better achieve teaching goals.

ii. This approach has a higher failure risk than the other approaches, since it may result in a completely new and untested course which will be presented to students as such.

iii. Teachers must take into account a large number of possible blended learning components and fully understand their implications. According to Walters (2008), a large choice of delivery mediums, a wide range of technological tools and the lack of examples to follow for particular combinations of activities, causes teachers to face complex decisions that need to be made and consequently to face pressure when redesigning the courses.

iv. It requires experience in blended learning design. Teachers who lack the necessary theoretical knowledge and experimental experience can hardly take full advantage of the potentials offered by blended learning. Knowing how to use technology and gradual experimentation with blended learning can be helpful to understand how technology can enhance teaching.

v. Planning and developing a new blended learning course is a lengthy process. Vaughan (2007) has established that developing such a course usually lasts two to three times longer than developing a similar course in the traditional format.

Recommendations

i. Teachers who have no designing experience in blended learning or have limited experience should first try one of the other two approaches in order to gain significant experience that can help them when applying this approach. Designing an effective course based on blended learning requires a considerable amount of knowledge and expertise necessary for the selection of the learning activities that are better realized in the classroom and others that should be transferred to the Internet (Walters, 2008). Mortera-Gutiérrez (2006) emphasises that one of the worst scenarios for a blended learning course is that the teacher does not use the most appropriate mediums to deliver the instruction.

ii. Teachers should be ready to invest considerable time (at least six months, but one year is the preferred option) in the design, i.e. the creation of the course. One of the greatest obstacles for the successful application of blended learning is the lack of time.

iii. Teachers need to consider the possibility of including a various mediums of content delivery in their courses. According to Carman (2005), e-learning is most effective when it employs a combination of different delivery options.

iv. Institutional support is the key factor for the success of this approach. According to Aycock & al. (2002), there has to be a high-level of institutional support in the form of time release, professional development, financing and technical support to the teachers.

Example. The chart below illustrates a possible transformation of (a part of) a traditional course by applying the high-impact blended learning approach.

Traditional (face to face) course			High-impact blended course	
1	lecture (face to face)		1	lecture (face to face)
2	lecture (face to face)	full redesign	2	pre lecture online quiz + lecture (face to face)
3	lecture (face to face)		3	google-hangout + face to face consultation
4	tutorial (face to face)		4	pre reading + tutorial (face to face)
5	tutorial (face to face)		5	onlajn test
6	test (face to face)			

Concluding remarks

Taking into account a large number of online components that need to be considered, teachers should undergo adequate training (professional development as lecturers) which focuses on the proper use of new educational technologies that they have not used before.

Low-impact blend	Medium-impact blend	High-impact blend
1. Teacher has no expirience in designing and developing for blended learning.	1. Teacher has designed and developed a blended learning course.	1. Teacher has several years of expirience in designing and developing for blended learning.
2. Teacher has no prior expirience in teaching the traditional course.	2. Teacher has tought the traditional course.	2. Teacher has made several iteration of teaching the traditional course.
3. Teacher has some knowledge in integrating technology.	3. Teacher has good knowledge in integrating technology.	3. Teacher has strong knowledge in integrating technology.
4. Teacher has no confidence in integrating technology.	4. Teacher has some confidence in integrating technology.	4. Teacher has high confidence in integrating technology.
5. No institutional support is provided.	5. Institutional support is provided.	5. High institutional support is provided.

Picture 4. Factors influencing the choice of approach in the course design

The main advice is that the teachers with no experience in blended learning design should start by using the low-impact approach, so when they gain more experience, they can move on to medium-impact approach. Only when they have gained sufficient self-confidence, knowledge and experience with blended learning are they ready for the high-impact approach. The ultimate goal should be the re-evaluation of the entire set of course goals bearing in mind what the students' needs are. This aim can be more easily accomplished by building the entire path from scratch. Such approach can provide the teachers with the opportunity start from a new perspective, re-examining the whole course and possible mediums of delivery which they will integrate in their courses, but first and foremost taking into

consideration the students' needs. However, the direct transition to high-impact blended learning is risky and can result in failure. Getting to know the technology and gradually experimenting with blended learning is the way through which the teachers gain an opportunity to become more self-confident and to properly realize the possibilities of enhancing the traditional teaching by using technology.

Flipped classroom

Flipped Classroom is a type of blended learning format and it is a teaching strategy whose goal is to increase student engagement and to stimulate successful learning through a particular model of using technology and applying active learning in the teaching process.

Flipped classroom, as a teaching format, is founded upon the idea of shifting the focus in the traditional teaching paradigm, so that the main activities of teaching and learning process, such as the classroom activities and homework activities, become flipped (switched) in relation to the traditional model. Acquiring the content takes place in the individual learning format, before the class, and the application of the acquired content and the problem-solving process take place in the classroom (Table 2).

	Before class	During class	After class
Traditional classroom		Acquiring the new content (teaching by the instructor)	Applying and enhancing the acquired content (homework activity)
Flipped classroom	Acquiring the new content (autonomous learning from video materials, selected texts…)	Applying and enhancing the acquired content (through activities during class)	

Table 2. Flipped classroom model in relation to traditional classroom model – comparative illustration of the activities in the traditional and flipped classroom

The first implementation of the flipped classroom model was recorded in the early period of XIX century, at the West Point American military academy. The teachers of the Academy introduced a teaching model that was based on the student requirement to study the teaching material before classes. During the class, in smaller groups, and through the application of the acquired concepts and information covered before class, they did problem-solving tasks together.

In the context of today' practice, individual learning that is required by the flipped classroom is mostly assisted by different technologies. Jonathan Bergmann and Aaron Sams, the chemistry teachers at the time in the secondary school of Woodland Park in Colorado, are considered to be the modern originators of this method that utilizes information technology. In 2000 (Bergman, 2012), while trying to resolve issues dealing with frequent student absences due to excessive extracurricular activities, they began to record their classes and upload them onto the YouTube platform, allowing the students access to the teaching material at all times.

Flipped classroom may include a myriad of extracurricular activities: observation of previously prepared recordings of teaching, studying the written material, research, homework, and the like. Since the general information and concepts must be present in the material that the student studies before the class, it is of extreme importance to choose the right method and type of teaching material that stimulate/ensure student engagement. The teachers are expected to be able to attract student attention and stimulate their motivation to engage in the process of individual learning by selecting the right model of presenting the content. Based on presenting the content in as specific way to enhance individual learning, the flipped classroom model enables the student to learn the content at an individual speed (Lopes, 2018.) The teaching activities may significantly differ from the traditional classroom, including the activities such as role play, debates, quizzes and group presentations. In comparison to the traditional teaching, the teaching models of the flipped classroom, especially those implemented during class, are usually connected to some form of group work and they mostly include cooperation among students (Picture 5).
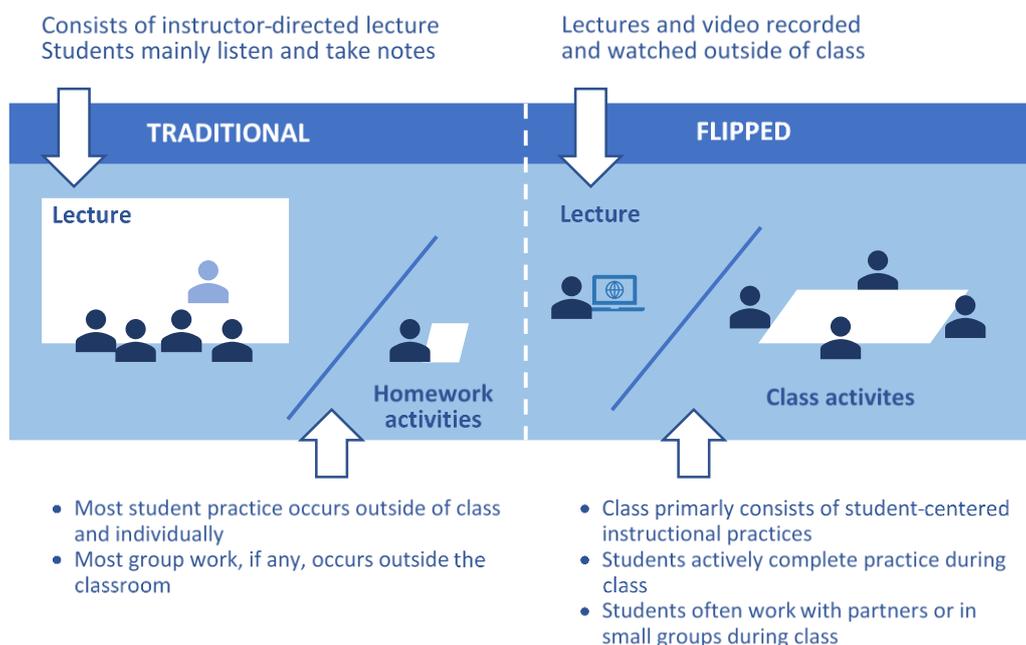
Picture 5. Traditional model in comparison to the flipped classroom model

When preparing the teaching-learning construct according to the flipped classroom model, the teacher must:

- Structure the pre-class student activities,

- Define the tools for presenting the material to the student,

- Create the teaching content and make it available for students,

- Plan out during-class activities that would stimulate active learning in students, assist them in detecting their mistakes in comprehension, assist them in finding the personal learning tempo and most importantly, to teach them how to learn,

- Structure interpersonal communication among students and stimulate peer-assisted learning.

The way learning goals are being met has been shifted in the flipped classroom in relation to the traditional classroom. If we take Bloom's taxonomy as an instrument of defining learning goals,

considering the fact that there are six levels of cognitive processes, from the least to the most challenging processes (remembering, understanding, applying, analyzing, evaluating, creating), the concept of the traditional teaching model mostly relates to the first three levels being met through the during-class activities, while in the flipped classroom model these levels are attained during the pre-class phase, at home. Cognitive process levels from 4 to 6, which require a more complex cognitive effort, are attained with the teacher support and during class in the flipped classroom model, whereas in the traditional classroom model they are more or less achieved in the post-class student activities (Picture 6).
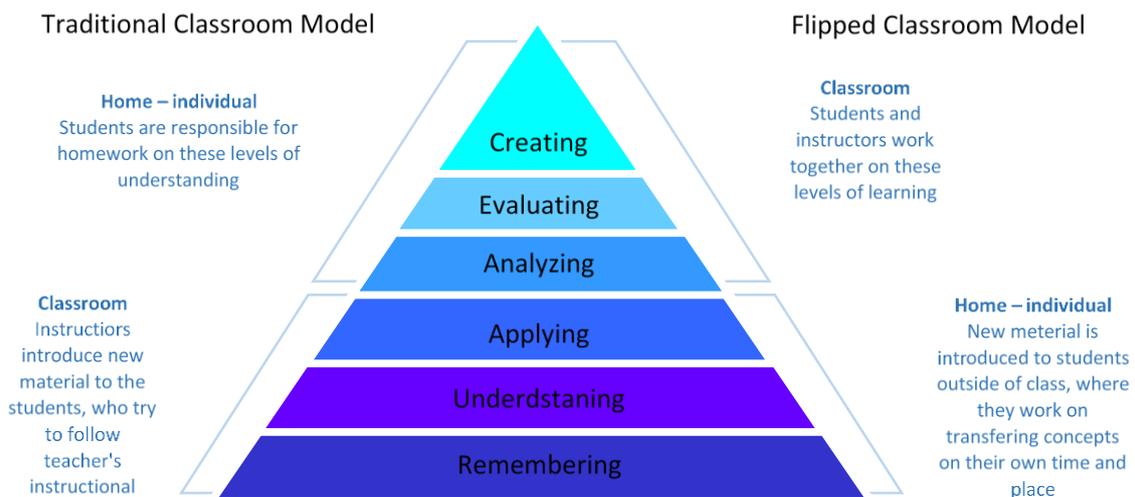
Picture 6. Achieving the learning goals categorized through Bloom's taxonomy cognitive levels in the traditional and flipped classroom model

The theoretical foundations of the flipped classroom model are usually established in the research related to student learning, with the main aim being the student active learning process, and the main type of learning being **active learning**. Characteristic methods of active learning in the flipped classroom are:

- Problem-Based Learning (problem-solving teaching)
- Peer-Assisted Learning, through Peer Tutoring and group work (Collaborative learning) or team work (Cooperative learning).

What is unanimous for these strategies is the shifted teacher role. In these activities **the teacher assumes the role of a team guide/manager.**
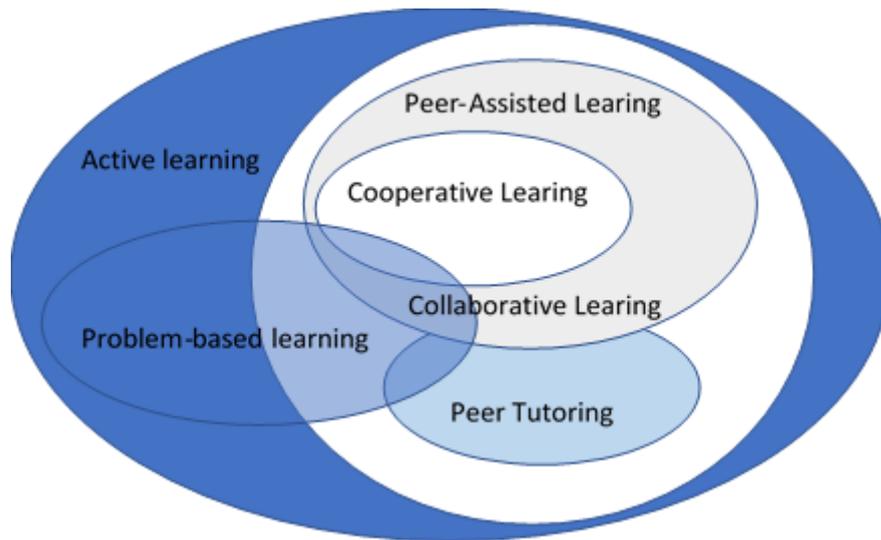
Picture 7. Theories and methods of student learning (Bishop, 2013)

Studies related to contemporary teaching competence requirements raise the importance of problem-solving and peer-assisted learning methods due to an increased need for professionals who can master the higher cognitive processes and team cooperation skills in new problem-solving situations (Bentlei, 2016).

Challenges in implementing the flipped classroom model:

i. For the implementation of activities out of class the students need to possess personal devices and have Internet access, which is not equally possible to ensure in all environments.

ii. It is not easy to inspect whether the student has studied/acquired all pre-class material.

iii. It is not an easy task to motivate students towards autonomous learning. The motivation is not the same or at the same level with all students and therefore the speed of successful acquisition of material is not the same. The issue may be related to planning the activities that would enable the students to learn at their own speed, and to simultaneously follow the teaching content and the rest of the group (Nielsen, 2012).

iv. For some of them individual student work outside the class is the same as homework not many are fond of. In addition, if the teacher does not properly anticipate the time needed for students to complete the task, and if at the same time an increased number of teachers also apply the flipped classroom model, student workload may easily be excessive.

v. Implementing the flipped classroom model requires the teacher to sufficiently master the needed technologies.

vi. The teacher must spend more time in preparing and organizing the entire process of activities. It is required of him/her to ensure an individual approach and means of following all student activities. Also, it is expected that the assessment, mostly formative, is varied and more frequent.

–Flipped Classroom model design

A careful preparation is essential for efficient and successful flipped classroom model. Every teaching unit must have a predefined lesson plan. The lesson plan of the teaching unit must define the following three components:

- **Outcomes**, by which the teacher answers the question *What do I want the students to learn?*

- **Activities**, by which the teacher answers to the question *Which teaching methods will I use?*

- **Assessment strategies for student progress and achievement**, by which the teacher answers the question *How will I assess the understanding of the learnt material?*
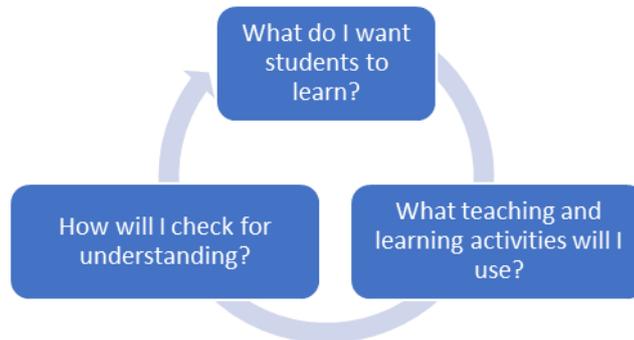
Picture 8. Key questions during the lesson planning

Outcomes

Before defining the outcomes, the teacher should answer the following questions.

- What is the topic of the lesson?

- What do I want the students to learn?

- What do I want students to understand and be able to perform at the end of class?

- What are the most important concepts/ideas/skills I want the students to acquire and why are they important?

Bloom's taxonomy is recommended so that the cognitive process the students are expected to perform could be described.

Example. When stating the learning outcomes, the teachers could use verbs describing different actions in Bloom's taxonomy: „At the end of the class the student will be able to…"

Level of Bloom's taxonomy	Actions
„Pre-class"	
remembering	define, state, remember, repeat, remind
understanding	summarize, explain, argue, demonstrate
„During-class"	
applying	use, demonstrate, apply, illustrate
analyzing	compare, examine, relate, categorize
evaluating	criticize, judge, review, defend, validate, test, argue
creating	construct, design, create, simulate, invent

Activities

Plan of the activities should include the order in which they take place (Picture 9):

- „pre-class",

- „during-class" and

- „after-class".
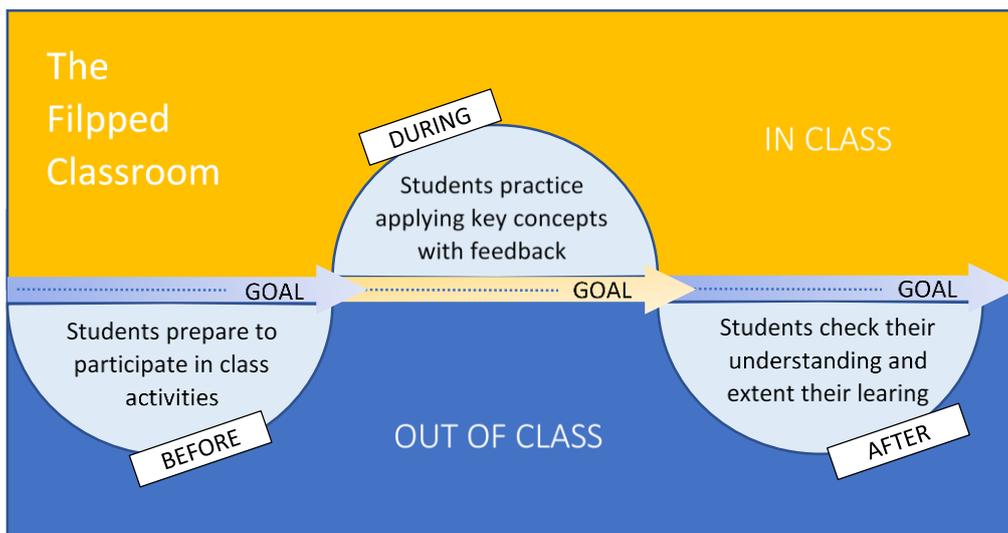
Picture 9. The activity order in the flipped classroom model

Regardless of the strategy, the activity plan allows the teacher to be able to answer the following questions.

- How should I explain the topic?

- In what ways can I illustrate the concepts presented?

- How can I interest the students for the topic and motivate them to engage?

- Are there adequate examples, analogies or situations from real life or from previous material which can help the students understand the new material?

- What do students need to do in order to understand the material better?

When we consider the flipped classroom strategy, it is especially important to consider the following questions.

- Is the material efficient and relevant for the presented topic?

- Do the students have access to the material?

- Do the students know how to use the material?

The pre-class activities have the goal to prepare the student for the activities during the class. Apart from defining them, it is necessary to decide how the teacher will provide the feedback to students on the issue of their success in acquiring the content, and related to the required level of acquisition needed for the successful involvement in during-class activities. This assessment does not only need to be a part of pre-class activities. On the one hand it can be designated for that teaching phase, with the aim of providing adequate feedback for the student. On the other hand, achievement assessment during class has the goal to assist the teacher in detecting problems in understanding, besides allowing him/her to record student progress.

The goal of planning during-class activities is the selection of activities that would ensure a more profound understanding of the topic and ability of students to perform higher-order cognitive processes. For all of the during-class activities it is necessary for the teacher to prepare clear instructions that will be distributed to students at the beginning of the class. In order to ensure consistent implementation and dynamics, it is necessary to create a timed order of activities.

After-class activities are not a requirement of the flipped classroom model, but depending on the topics covered, they may be of importance. Their aim should be to ensure long-term retention of the content, skills and perspectives. In addition, they may be useful to introduce the topic of the upcoming class.

Example. Example of the timed order of during-class activities (Hartyányi, 2017)

Timeframe	Parts of the lesson	Teaching strategy		
		Methods	Working forms	Tools
10 min	Checking what has been learned in the study of teaching materials	Gamification	Individual. The whole class	Kahoot
15 min	Selection of jobs to be worked with		Individual	Notice board with profiles
5 min	Group of students based on the selection of postitions		Group (cooperative) Roles: Coordinator: organize work and boost work	Countdown timer
50 min	Definition of training actions and their didactic programming	Case study	Secretary: write relevant information, save and share documents Controller: time and task compliance Spokespearson: speaks on behalf of the group to consult doubts to the teacher	One laptop per team
25 min	Presentation of work done			Given
15 min	Evaluation of the activity and the work done		Individual and group	Evaluation tools: survey, rubric and checklist

Monitoring/assessment of the student progress and achievement

Assessment is aimed at testing the fulfilment of the defined outcomes. It is important to consider which tools can be used to determine whether each of them has been achieved. The teachers should define models of assessment, together with the criteria, standards of assessment, as well as the ways of providing feedback for students.

One of the important traits of the flipped classroom strategy is the student-focused approach and it should be applied during the process of assessment. In the contemporary teaching practice, a constant evaluation of student progress and achievement is required. In the student-focused approach evaluation relates to continual monitoring of the student and regular access to feedback about the learning process. In accordance to the planned activities the following forms of evaluation can be applied, dependent upon who the evaluator is and whose work is being evaluated:

- review of the individual achievements of each student by the teacher,

- evaluation of group activities by the teacher,
- self-evaluation,
- peer evaluation.

During the implementation of the flipped classroom model, evaluation is an integral part of the process of teaching and/or learning, with the aim of advancing the assessment for learning.

Inadequate preparation of students for their cooperation in the during-class activities may seriously impair the efficacy of the flipped classroom. For that reason it is of utmost importance to plan the formative assessment that should be an integral part of the pre-class activities, the so-called *pre-formative assessment* (Hartyányi 2017, Persky 2017). This assessment is applied when the students have independently acquired the new material, and before they engage in any type of group interaction. Pre-formative assessment provides a reliable perspective about what the students have learned before class or about any group activities. Also, for the assessment plan, the teacher should include personalized assessment such as:

- using the material outside of the classroom,
- adequate application of concepts in different contexts,
- active involvement in real-life communication,
- cooperation with other students.

–Tools of the Flipped Classroom

Technologies that may be used for the implementation of the flipped classroom model, and which will be presented below, will be divided into four categories:

1) Video-creation tools,
2) Video-uploading tools,
3) Video-interaction tools,
4) Learning Management Systems.

Video-creation tools

Independent creation of video materials does not need to be expensive, nor time-consuming. The equipment needed which is today most frequently a part of the standard configuration of the PCs and which the common users possess:

- microphone,
- web or video camera,
- writing board or smart board (if the institution has one) – depending on the field of the course and the nature of the content and tasks, the board may be necessary to a higher/lower degree.

For the recording of the content itself there are two types of software tools that can be used:

- The tools for recording events over the entire or one part of the screen. Most frequently they have an ability to record the audio from the microphone, as well as recording the sound of the system itself.

- The software for recording the video from the camera. These types of software tools can store the video recording in a special window and synchronize it with the recording of the screen.

There are numerous tools that provide simple recording of the video material. Some of the most popular have been presented below.

- *Screencast-o-matic*

This is a web platform that allows recording of the screen directly from the web browser, recording of the camera and microphone signal, as well as 'picture-in-picture' imbedding and so on. It also has the postproduction support.

It is possible to use it without paying the fee, with specific limits in the tool use. It requires registration and then the installing of the tool for screen content and so on. The free version allows the recording of the material not longer than 15 minutes. Instructions manual: https://screencast-o-matic.com/tutorials

- *Camtasia Studio*

This is a commercial software tool that allows recording of the screen or some of its part, as well as the camera recording, which can be set into a 'picture-in-picture' mode. It is possible to record from the MS Power Point presentation.

It allows for the video processing, inclusion of multimedia content, clouds, animations and application of different effects for the video and audio recording. Instructions manual: https://academy.techsmith.com/

- *MS Power Point*

MS Power Point tool has a free add-on that allows recording of the slides and adding images from the camera. Recording of the screen is also possible. Instructions manual: https://support.microsoft.com/en-us/office/record-a-presentation-2570dff5-f81c-40bc-b404-e04e95ffab33

Uploading video services

Uploading of the videos is possible through different free uploading services available on the Internet. The most important uploading service is the *YouTube*, and within the MS Office 365 Suite there is the *MS Stream*. In addition, most of the platforms for online learning support have the video material uploading option.

- *YouTube*

This is the uploading and sharing service for the video recordings. It supports a wide range of formats for uploading videos: .mov, .mpeg4, .mp4, .avi, .wmv, ...

It is possible to upload recordings of up to 15 minutes without special requirements. It is possible to upload longer recordings if certain requirements are met, which is sufficient enough for the teacher's needs. There is a support for playing the video in different resolutions, as well as the support for live streaming.

Uploading instructions manual: https://support.google.com/youtube/answer/57407

- *MS Stream*

This is a platform that allows video material streaming from the web browser. It is accessible to registered account users. It allows uploading, editing, viewing and commenting of the video material. Instructions manual: https://www.youtube.com/watch?v=3oKMXC9xhe0

Tools for creating interactive videos

Interactive video materials are those that include video material supplemented by elements that allow interaction with students. There are various applications that allow this format:

- *EdPuzzle*

This is a free application that enables uploading of the video material and its modification for teaching. It is possible to cut the video, to add questions, quizzes, comments, as well as audio content. The platform offers a great number of already uploaded video materials, available for use as well as for additional modification.

Using his/her own account the teacher defines the course for which the material is created before uploading it onto the platform, making it available to students through access codes. The teacher also gets reports of student activity. Instructions manual: https://support.edpuzzle.com/hc/en-us/categories/360000701132-For-Teachers

- *PlayPosit*

This is a web application that allows adding of interactive elements into the video materials that have been uploaded in video streaming services such as YouTube and Vimeo. It is possible to add questions, tasks and quizzes for comprehension check of the content in the video recordings, as well as following the activities and student achievements. Instructions manual: https://knowledge.playposit.com/category/209-playposit-30

Learning Management Systems

Learning Management Systems – LMS are software systems that allow virtual, collaborative environment for the support of the teaching process. Learning Management Systems provide support in:

- Uploading teaching materials in different formats (text, images, audio and video recordings) and their organization in different forms, such as blogs, forums and traditional web sites, and so on
- organizing of online testing and homework tasks in different formats,
- student activity and achievement follow-up,
- teacher-student communication, one-on-one and group, and communication among the students,
- access from any place and at any time,
- real-time material updating.

Moodle and *Canvas* are some of the most important LMS systems, but apart from these there is a myriad of popular systems that more or less allow the organization and management of the teaching material and following of student activity: ***Blackborad, Piazza, EdModo, Google Classroom, MS Teams.***

- *Moodle*

This is a free system for managing the open-source teaching. To install and use it, it is necessary to provide an available server and administrative support for the installation.

It allows a wide range of tools for creating and managing the content, following pstudent progress through tests, quizzes and tasks, managing the users, user roles and groups of users. It is possible to use forums, wiki and chat for the student-teacher communication.

This platform is widely used in many countries, as well as in Serbia, both at the universities and in the secondary school system. It has been translated into more than 100 languages. More about the platform: https://docs.moodle.org/310/en/Main_page

- *Canvas*

This is a free platform and only registration is necessary for its use. For every course created on his/her account, the teacher must publish it so that it can be made available for the students. It is possible to post notices, tasks, discussions, tests and quizzes. This platform is especially popular in university courses worldwide. More about the platform: https://beaver.instructure.com/courses/670/pages/welcome-to-canvas-for-beginners

Remarks on creating video material

Creating quality video materials requires an expert teacher, or at least the one who is ready to invest time to acquire experience in using technology for creating video materials. For many issues there are very valuable video materials available on the Internet. The teacher's task is to find them and to evaluate whether they are adequate for lesson planning of a specific teaching unit.

There are many crucial suggestions the teachers should abide by in creating educational video materials for the students (Bergman, 2012).

- **Short form**

The video duration should not be one or two classes long, but in fact it should have a short form, not longer than 10-15 minutes. It is suggested that only one topic/concept/procedure should be covered by the video. Limiting of the time requres the focus on the most important facts and examples. By no mean does that involve that the teaching content should be minimal, but that it is necessary to define smaller and clearly defined portions of the content that will be developed in one video.

- **Speech dynamics that stimulate the listener**

The main teaching material presented in the video class is usually a presentation or some other form of static content, which by itself is not attention-grabbing for the listener. The dynamics is above all determined by the lecturer's talk and activity such as note-taking, object appearance, animations by which some process is presented and suchlike. When we speak of lecturer's talk, the dynamics depends on the change of tone and voice of the speaker.

- **Including the lecture interlocutor**

Including an interlocutor in the presentation so that one interlocutor takes up the role of an expert, and the other the role of a student, probing questions and presenting most common student dilemmas, may be of great assistance in acquiring and understanding the content.

- **Anecdotes and witty comments**

Witty comments, pictures and anecdotes may help in the attention-grabbing process.

- **Stick to the topic**

Every example or story must be justifiable and must have a function of understaning the problem, the issue being explained etc.

- **Note-taking during presentation**

The use of a digital pen for writing the formulas, selecting and noting crucial facts in the presentation attracts the attention of the listeners.

- **Including descriptive clouds**

The object, text or a graphic design that appears next to/over the screen recording attracts the attention of the student. Noting the important facts during the lecture is possible through the use of objects that appear during the video at the time when it is important to make inferrences, to emphasize or repeat an important section.

- **Zooming parts of the screen**

The postproduction tools make it possible to enlarge parts of the recording that are important for the ongoing lecture and by that to additionally attract student attention to important parts of the content being explained at the time.

- **Pay attention to the user rights and the distribution of authorized material**

–Examples of the Flipped Classroom

We will illustrate two examples of the application of the flipped classroom model, one of them being related to teacher training, i.e. professional advancement course, and the other is related to a university course. The role of these illustrations is to present previously mentioned theoretical descriptions and to provide additional ideas for conducting other university courses based on a similar model.

Example 1. – Teacher training for video material use. By presenting an example of a pilot project of teacher training for planning and creating video teaching material, we will illustrate how the model of the flipped classroom has been used (Hartyányi, 2017).

The teacher training lasted for 5 weeks. The topics covered are presented in Table 3.

Week 1	Week 2	Week 3	Week 4	Week 5
Pre-production	Shooting Scenarios Introduction to the camera	Learner Engagement Multimedia Principles Use of the Camera	Production Skills Editing	Publishing

Table3. Teacher training for planning and creating video teaching material – weekly scheduled topics

At the beginning of each week the trainees were delivered 40-minute lectures, which also included tasks to be performed in the duration of 10 minutes.

Twice a week live 1-hour workshops were organized and the dynamics of the workshops is presented in Table 4.

Timeframe	Parts of the lesson	Working forms	Tools
5 min	An overview of project progress	The whole class	Presentation/Kahoot

10min	A quick recap of the latest video lesson(s)	The whole class	Video
15 min	Open discussion and debate	Group Led by instructor	Mindmap
10 min	Collaborative opportunities	The whole class	Brainstorming tools
15 min	One on one discussion & problem solving	Individual	N/A
5 min	Any additional project updates	The whole class	Evaluation tools/ Datagathering tools

Table 4. Teacher training for planning and creating video teaching material – dynamics of workshop activities

The evaluation was based on reporting the trainee progress each week, based on the lecture topic being taught. Therefore, for example, in the 1st week, where the topic was pre-production, the participants had the task to submit a story and/or scenario, while during the 2nd week, which was devoted to using a camera, the participants were required to submit a recording they have made as well as a short text about a personal reflection on the progress and the process itself.

Example 2. – University course. We will show how the flipped classroom model was applied based on the example of a university course of Web programming at the University of North Carolina. Over the years teacher experiences on course organization have been collected in Maher (2015).

The course is followed by 60 to 90 participants. The topics being covered include programming languages for the development of web services (html, css, javascript i jquery), as well as the topics such as the design of these services. The entire course has been conducted in a form of a flipped classroom model since the year 2013.

The weekly dynamics of the course includes independent student work before class and small group work during class. Weekly group activities are devoted to lab activities, where the plans and goals of the activities are made available to students before the activities (Table 5).

Pre-class activity includes introduction of the content that is provided for the students in the form of video recordings and texts, while during-class activities are not meant for lectures or repetition of the topics distributed during pre-class phase. It is expected of students to be able to independently try out the codes of the explained programming languages.

The post-class activities include tasks such as searching for the web pages the covered topics are demonstrated, drawing of conceptual diagrams and writing of pseudocodes. These activities are conducted in groups. Due to the fact that initial student requirement is to develop some programming skills individually, the activities and discussions during class are comprehensible and less abstract. The use of Moodle platform for follow-up and communication has enabled the insight into the fulfilment of these requirements. A schedule that is clear and accessible to students has been defined for all of the

pre-class activities, so the students are prevented from participating in the specific activities, such as quizzes, if the previously determined activities have not been labelled as complete.

One of the major during-class activities is a lab task. During each class students receive tasks that they have to solve in pairs. At the beginning of the course the activities are usually comprised of changing and expanding the given code, where during the progress of the course it is expected of them to write the code from the scratch. The skills and knowledge necessary for task realization are aligned with the topics designated for that week. Lab activities are planned in such a manner that if the student is prepared for the class, he/she will be able to meet the requirements in the previously determined time period. Organizing the student pairs that will work together changes weekly or biweekly. An integral part of every class is a quiz. These quizzes are used in the different manner as an activity that stimulates peer-assisted learning, but also that enables detecting and dealing with mistakes in comprehension, in the following manner:

- Every student does their own quiz sample. The quizzes are exchanged among students for assessment. The teacher is involved in the analysis of the grades through the discussion about the reasons for the (in)correct answer to each question.

- The student fills out the quiz individually, and then he/she discusses the correctness of the answers with the students next to him/her until all the neighbouring students have the same answer. Every student is assessed individually.

- The students are divided into groups. For every question the group must agree on the correct answer, after which the defined group answer is recorded in the group quiz. All of the students in the group receive the same grade.

- The students participate in the online quiz (*clickers*), while they are allowed to communicate. The teacher shows the quiz statistics and can repeat the assessment. For assessment and grading the speed and correctness of the answer are compared. Allowing for the competition context in these quizzes increases student engagement and motivation.

- The students are divided into groups. Each group receives a video material and based on the video content they create a quiz. The groups interchange the quizzes for solving, while the quiz designers grade the answers.

Regardless of the means of conducting the quiz, the focus is on peer-assisted learning and detecting the mistakes in comprehension of content (terms, concepts, claims, procedures…). If there are issues in understanding, the teacher assists with additional clarification on the spot, or conducts a brief topic-focused lecture. Due to these reasons quizzes usually are significantly time-consuming for the class.

Student opinion is highly regarded during this course. Student surveys are conducted during the course and at the end of the course, with the aim of obtaining feedback related to the course conduct. In open-ended surveys the students have stated that in relation to the traditional teaching paradigm they were more involved in the teaching process, and that the communication and collaboration with other students were significantly increased. The student comments were mostly positive and they illustrated that the students enjoy being involved in the activities that are focused on hands-on theory. For example, some of the comments were: "I liked the fact that there are more hands-on activities than in other classes" and "I enjoyed the fact that lab activities actually helped me learn in-class and the fact that all of the videos were useful". When addressing the question what they didn't like in this teaching model, the students indicated that they needed more time for the lab activities, and that the quizzes before the

lab activities were not welcome. A small percentage of students reflected on this type of teaching model as a negative one. Their reasons for suchlike standpoint were: "We were assessed on the content that was not covered in class later on" and "Watching the video recording I had an impression that I was following an online class". Suchlike comments were a result of lack of understanding for the flipped classroom model, which indicates that introducing the students to this type of work and goals of this teaching model is of extreme importance, and therefore we need to pay more attention to it, because complete understanding will ensure adequate student motivation. A full insight into the student survey results may be found in Maher (2015).

References

Amiel, T., Orey, M. (2006). Do you have the time? Investigating online classroom workload. *Journal of Educational Technology Systems*, 35, 31-43.

Alammary, A., Sheard, J., & Carbone, A. (2014). Blended learning in higher education: Three different design approaches. *Australasian Journal of Educational Technology*, 30(4), 440- 454. https://doi.org/10.14742/ajet.693

Aycock, A., Garnham, C., & Kaleta, R. (2002). Lessons learned from the hybrid course project. *Teaching with Technology Today*, 8(6), 9-21. Retrieved from http://www.uwsa.edu/ttt/articles/garnham2.htm

Barbour, M. (2014). History of K-12 Online and Blended Instruction Worldwide: Handbook of Research on K-12 Online and Blended Learning. N.P., ETC Press Publ., pp. 25–50.

Bergman J. & A. Sams (2012). *Flip Your Classroom. Reach Every Student in Every ClassEvery Day*, International Society for Technology in Education, USA

Bishop, J. L., & Verleger, M. A. (2013). The flipped classroom: A survey of the research. Paper presented at the 120th American Society for Engineering Education Annual Conference and Exposition, Atlanta, GA, 23-26 June 2013 (pp. 23.1200.1 - 23.1200.18)

Bloom, B. S. (ed.) (1956). Taxonomy of Educational Objectives: The Classifica- tion of Educational Goals. Handbook I: Cognitive Domain. New York: Longmans, Green.

Carman, J. M. (2005). Blended learning design: Five key ingredients. Learning Technical Report. Agilant.

Clark, R. C. (2002). The New ISD: Applying Cognitive Strategies to Instructional Design. *Performance Improvement Journal*, 41(7). https://doi.org/10.1002/pfi.4140410704

Duhaney, D. C. (2004). Blended learning in education, training, and development. *Performance Improvement*, 43(8), 35-38. doi:10.1002/pfi.4140430810

Ertmer, P. A, & Ottenbreit-Leftwich, A. T. (2010). Teacher technology change: How knowledge, confidence, beliefs, and culture intersect. Journal of Research on Technology in Education, 42, 255-284. doi:10.1080/15391523.2010.10782551

Harriman, G. (2004). Blended learning. Retrieved from http://www.grayharriman.com/blended_learning.htm#1

Hartyányi M. at all (2017). *Flipped classroom in practice*. https://www.flip-it.hu/en/system/files/konyvek/flipit_book_en.pdf

Hofmann, J. (2006). Why blended learning hasn't (yet) fulfilled its promises. In C. J. Bonk & C. R. Graham (Eds.), Handbook of blended learning: Global perspectives, local designs (pp. 27-40). San Francisco, CA: Pfeiffer Publishing.

Kaleta, R., Skibba, K., & Joosten, T. (2007). Discovering, designing, and delivering hybrid courses. In A. G. Picciano & C. D. Dziuban (Eds.), Blended learning research perspectives (pp. 111-143). Needham, MA: Sloan-C.

Lee, S., & Lee, H. (2008). Professors' perceptions and needs on blended e-learning. Paper presented at the World Conference on E-Learning in Corporate, Government, Healthcare, and Higher

Education 2008, Las Vegas, Nevada, USA. Retrieved from http://www.editlib.org/p/29737

Littlejohn, A., & Pegler, C. (2007). Preparing for blended e-learning. Abingdon, Oxon: Taylor & Francis.

Maher M. L., Latulipe C., Lipford H. & Rorrer A. (2015). *Flipped Classroom Strategies for CS Education*. In Proceedings of the 46th ACM Technical Symposium on Computer Science Education (SIGCSE '15). Association for Computing Machinery, New York, NY, USA, 218–223.

Mortera-Gutiérrez, F. (2006). Faculty best practices using blended learning in e-learning and face-to-face instruction. International Journal on E-learning, 5, 313-337. http://www.editlib.org/j/IJEL/

Lopes A.P. & Soares F. (2018) *Flipping a mathematics course, a blended learning approach*. 12th International Technology, Education and Development Conference, Valencia, Spain, 5-7 March, 2018 (pp. 3844-3853)

Persky AM, McLaughlin JE (2017). *The Flipped Classroom - From Theory to Practice in Health Professional Education*. Am J Pharm Educ. 2017; 81(6):118

Polja, Dj. (1966). Kako ću rešiti matematički zadatak?, Školska knjiga, Zagreb

University of Texas at Austin (2021). *Flipped Classroom*. Retrieved from https://facultyinnovate.utexas.edu/instructional-strategies/flipped-classroom (Accessed: 25/04/2021).

Vaughan, N. (2007). Perspectives on blended learning in higher education. *International Journal on E-Learning*, 6, 81. http://www.editlib.org/j/IJEL/

Zemke, R. (2002). Who Needs Learning Theory Anyway? *Training Magazine*, 39(9), 86-88.

Walters, B. (2008). Blended learning-classroom with on-line. The CALSCA Online Magazine. Retrieved from http://calsca.com/Writings/walters_blended_learning.htm

Wozney, L., Venkatesh, V., & Abrami, Pp. (2006). Implementing computer technologies: Teachers' perceptions and practices. Journal of Technology and Teacher Education, 14, 173-207. http://www.editlib.org/j/JTATE/

Authors:
Nebojša Jasnić
Siniša Đurašević

TOPICS:

Teaching methods in biology

From traditional to interactive learning at the University of Belgrade-Faculty of Biology

Application of interactive virtual solutions

Review of the TeComp project contribution on teaching modernization

Modernization of teaching and learning biology

Content

Introduction

Biology is a complex science whose study/learning in some individuals covers the period from the beginning of general education to the end of the academic career. During this period, we encounter different needs and goals of learning biology, and thus different learning methods.

One of the international goals related to the future of humanity is to build a sustainable society, which implies such a development of humanity in which the current generation of people meets their needs without endangering any future generation to do the same. (Brundtland, 1987). This goal necessarily includes the ecological, economic and social dimension. Since environmental challenges are an integral part of biology, it is necessary to pay special attention to learning biology from general primary education onwards. This problem has been recognized by some countries before others, so Finland, Sweden and Denmark, for example, are considered the founders of modern methods of learning biology. There are not many studies in the literature that deal with the assessment of the importance of teaching methods in biology (Jeronen et al., 2017), but they certainly differ depending on whether biology is studied within general education or at the academic level, whether teaching takes place in nature or in the classroom/laboratory, whether groups of students are larger or smaller,… That is why in the literature it is possible to find papers dealing with teaching methods in physiology (Rehan et al., 2016, Santhakumari Nagothu et al., 2016, Sri Nageswari et al., 2004), cell biology (Veselinovska et al., 2011), ecology (Jeronen et al., 2017),…

In this text, we will try to emphasize some problems that teachers face at all levels of biology teaching, but also suggest solutions, given the modern approach to learning this subject and the problem imposed by the current situation, and that is work on distance.

Teaching methods in biology

Teaching methods can be viewed as goal-oriented activities, as well as the flow of information from teachers to students. The study of teaching methods is important because they affect all types of learning, both cognitive, affective and psychomotor (Karami et al., 2012). Traditional biology teaching is deductive, and combines the methods and principles of instruction that the teacher uses to achieve the desired process of learning and/or memory in the student. In such an approach to learning, with the teacher at the center, teachers are the authorities and students have a role in passively receiving information through lessons and direct instructions. The alternative teaching approach is inductive and

the instruction begins with observation, experimental results to be explained, or a real problem to be solved. In this approach to learning, with the student at the center, the teacher and students have an equally active role in the learning process. The main role of teachers in this process is to guide students, facilitate the process of learning and understanding the material (Prince and Felder, 2006). The choice of teaching method usually depends on what type of teaching approach the teacher prefers, and less often on the learning goal itself.

In the teaching of biology, the selected teaching methods should enable the learning of biological facts, mastering practical work, as well as learning about biology as a science. In addition, several biological topics require approaches that include solving experimental problems and developing the skills necessary for independent work and the organization of practical work (Keselman, 2003). Since the research process in science implies the engagement of higher cognitive levels, it is assumed that students at that level already possess a certain degree of basic (factual) knowledge, as well as to have the ability to learn independently. Therefore, in the learning process, it is important to implement methods that include both, independent research and directed (instructed) teaching. The implementation of a problem-based learning model has a positive impact on students' academic achievement as well as their attitude towards individual courses (Akinoglu and Tandogan, 2007). Besides, the implementation of problem-based learning combined with group work encourages students to think critically through planning, discussing, asking questions and defining problems, as well as proposing solutions (Asyari et al., 2016).

Teaching activities conducted in the field provide students with interactive and authentic experiences, as well as the opportunity for experiential learning, which increases students' attention and contributes to learning efficiency (Simmons et al., 2008). Work at the field gives students the opportunity to observe nature and their environment and thus test the ideas and concepts they have learned in the classroom.

From all the above, it can be concluded that modern biology teaching is increasingly using interactive methods of working with students, putting students at the center of the teaching process. This is confirmed by Jeronen and associates (Jeronen et al., 2017) in a review paper in which, among other things, they studied which teaching methods are most common in the teaching process related to sustainable development from the aspect of biology.
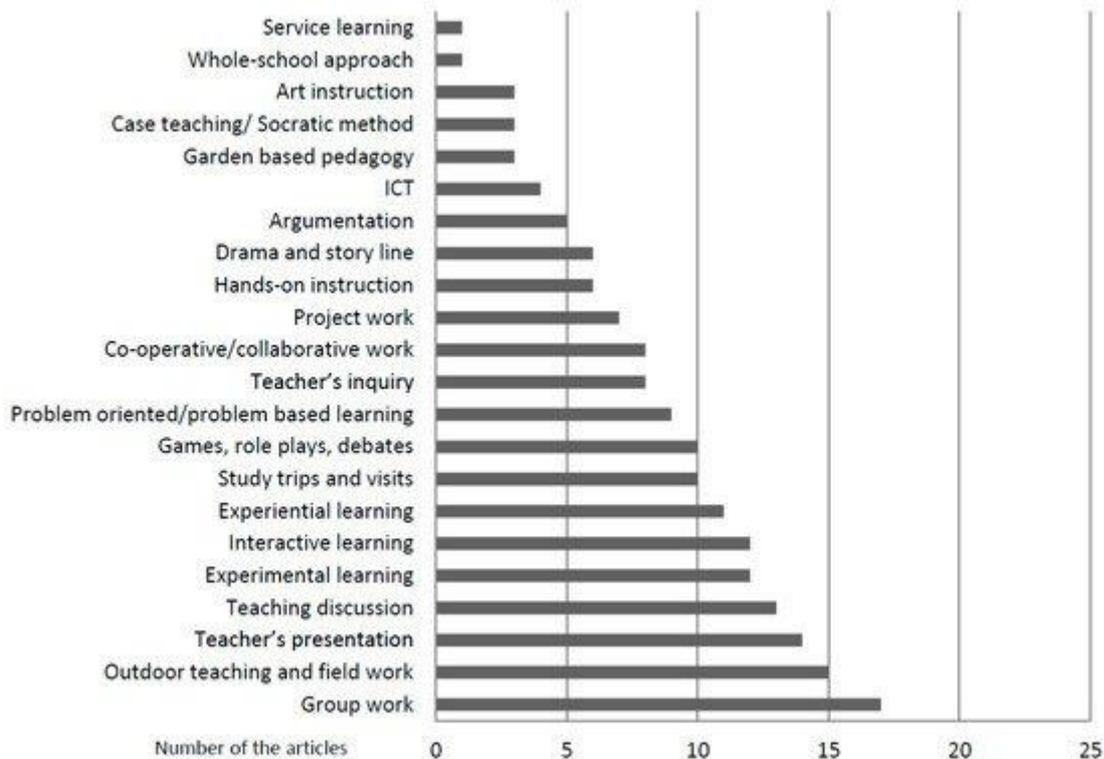
Figure 1. Representation of teaching methods in the teaching process related to sustainable development from the aspect of biology. Retrieved from: (Jeronen et al., 2017)

The graphs show that the most commonly used method is group work, followed by field work, experimental learning, interactive and experiential learning. Teacher's presentations and discussions are also popular and have been used mainly to introduce students to the way and goals of work. Such an approach is in line with papers that suggest that active learning has a positive effect on memory consolidation (Grant, 1997), encourages motivation to learn (Kern and Carpenter, 1986) and develops practical skills. Field work also has a positive effect on long-term memory due to the connection of experience with learned facts.

Problem-based learning, cooperative learning and argumentation as teaching methods were used in at least 1/5 of the studied papers. Argumentation encourages critical thinking and if used in conjunction with collaborative work, encourages students to pay more attention to a given problem (Driver et al., 2000).

Whole school approach and service learning are less common methods but should not be neglected as they can provoke public interest and encourage local, state and other influential services to support selected goals in education.

From all the above, it can be concluded that a modern approach to teaching biology involves interactive learning, with the student at the center of the process and the teacher who has an equally active role in directing, encouraging, creating problem situations and scenarios, but also instructing and training. There is resistance to this type of learning in our curricula, so often the students themselves are reluctant to interact, fearing that they will be "embarrassed" if they publicly give an incorrect answer or a wrong idea. However, having in mind the numerous works that support the idea of interactive learning as a modern approach to teaching, it is justified to insist on such a transition.

The following are examples of this transition from the practice of the Faculty of Biology, University of Belgrade, where we will try to show that resistance to change is a universal companion, but that the potential benefit is that it should prevail in teaching planning and method selection.

From traditional to interactive learning at the University of Belgrade-Faculty of Biology

Theoretical classes in most subjects at the Faculty of Biology, University of Belgrade are still held in the traditional way, in the form of lectures that students listen to, after which the teacher conducts an examination and gives an assessment according to previously established and known criteria. This means that students generally, despite the numerous obligations from the given subject (practical exercises, theoretical classes, colloquia,…), still invest a lot of effort and time to prepare the final exam in oral form, often long time after attending classes in that subject. As a result of such an organization, passing the exams is generally mediocre and a large number of students struggle to keep up with current obligations and backlog exams.

As teachers are not obliged to adapt teaching to the needs of students or modern trends, teaching methods mostly belong to those where the teacher is at the center of the process and students passively receive information. If a teacher recognizes the problem of student passivity during teaching, it is still a problem because there are practically no courses that the teacher could attend in order to improve the quality of teaching. Over time, this has generally discouraged even the most ambitious teachers, and the transition has been very slow.

Here we can look back at the experiences of one of our colleagues who recognized the problem and was persistent enough to look for a solution. The solution has been described in detail in the literature (Vujovic, 2016). In short, Professor Vujović, recognizing the problem, applied for a scholarship from the Fulbright Faculty Development Program and, after the scholarship was approved, he began his training at the College of Natural Sciences of the University of Texas at Austin, TX. There he noticed that the teacher, if he wants the class to be interactive, must instruct the students how to prepare for the upcoming lecture. In this way, students independently acquire basic knowledge before entering the class. To make sure that the students have completed the task, the teacher gives them short questionnaires whose solution is part of the final grade. Thus, the teacher is able to direct students to higher cognitive processes in class, choosing appropriate teaching methods that do not fall into the field of mere information transfer. This is the responsibility of the teacher who must devise activities that will encourage more complex forms of thinking in students, such as analysis and assessment. As we have seen before, these are all methods that aim to consolidate knowledge while stimulating student curiosity and interest in the subject. One of the applied methods can be a discussion among students, before deciding which answer to give to the question. Furthermore, it is possible to ask questions with multiple choice answers, solve problem-based situations, interpret selected tables, graphs, compare different biological processes, etc.

In order to motivate students to a proactive approach to teaching, especially in an environment that is often associated with a traditional approach to teaching, it is useful to prepare a list of goals and tasks that remind students how their activities are evaluated and how they participate in final assessment. It turned out that students initially resist this approach, probably because they are used to

coping with a different system, in which, regardless of all individual activities during the semester, they still have to prepare for the final exam. Given this, we can assume that it is easy to lose sight of the advantage of constant work that is evaluated in an appropriate way. Thus, we come to the conclusion that in our environment, one of the most demanding goals of teachers is to stimulate students to be active throughout the semester. What are the concrete results of such an approach to teaching at the Faculty of Biology, University of Belgrade? First, there has been an increase in the number of students attending lectures (from about 3% to over 80% of all students who have registered for the course)(Vujovic, 2016). Furthermore, the number of students coming for consultations increased to the extent that it was necessary to organize group consultations that were held in regular periods. In the end, about 90% of students successfully completed the course by the end of the current semester, which is a significant improvement over the former about 30%. However, about 7% of students failed to get used to the new way of teaching, learning and testing.

Professor Vujović in his work (Vujovic, 2016) concluded that the transition from traditional to interactive learning is a valuable experience, both for him and for his students. It should be mentioned that among fellow teachers at the University of Belgrade, there is great interest in the knowledge and experience that Professor Vujović gained in the United States, so that numerous lectures on this topic were held, and some teachers already include some of the proposed activities in their lectures. The author of this part of the text joins such a conclusion, considering that he himself had an insight into the way of work and thinking of students who previously attended the course of Professor Vujović.

Having in mind everything that has been said so far, we come to the conclusion that the modernization of teaching is necessary and that both teachers and students can benefit from it. Students will acquire more complex knowledge and will be free from fears of debates and discussions, and teachers will be more challenged to work in such a proactive environment. Also, having in mind another challenge that has recently significantly affected the implementation of all teaching activities, regardless of the type of educational institution (work at distance), in the next part of the text we will try to approximate the use of information technology in teaching biology.

Application of interactive virtual solutions

Upotreba informacionih tehnologija u nastavi biologije pratila je sam razvoj neophodnog hardvera i programskih paketa pa tako još pre 40tak godina nailazimo na prve radove koji se bave ovom temom (Katz et al., 1978). Not counting the very complex computer methods for predicting various phenomena in biology, which facilitate the work of scientists in the field or in the laboratory and lead to new scientific knowledge (Dortel et al., 2020), today, there are numerous software packages that can simulate biological phenomena in a simple but satisfactory way and thus help teachers to enrich biology teaching in an interactive and interesting way.

One of these software packages has been used in the teaching of physiology/endocrinology at the Faculty of Biology, University of Belgrade for ten years. It is "PhysioEx: Laboratory Simulations in Physiology", a software package that has been available for many years (Zao et al., 2005). The version of the software package that we started using is PhysioEx v6.0 and consists of 13 modules and 40 laboratory simulations in the field of physiology, which can be used as an add-on or replacement for laboratory work. The software package, which we will explain in detail in this text (PhysioEx v8.0), is

The software package consists of 12 modules:

1. Cell transport mechanisms and permeability,

2. Skeletal muscle physiology,

3. Neurophysiology and nerve impulses,

4. Endocrine system physiology,

5. Cardiovascular dynamics,

6. Cardiovascular physiology,

7. Respiratory system mechanics,

8. Processes of digestion,

9. Renal system physiology,

10. Acid/base balance,

11. Blood analyses,

12. Serological testing

and additional material in the form of a histological atlas and individual explanations. Each module is divided into several individual experiments and material that gives students a theoretical basis for the problem to be processed experimentally, as well as a short questionnaire that students fill out after the experiment.

Consider, for example, the module Endocrine system physiology. In addition to the introductory material, this module contains the following experiments:

1. Metabolism

2. Hormone replacement therapy

3. Insulin and diabetes – Part 1

4. Insulin and diabetes – Part 2

5. Measuring cortisol and andrenocoricotropic hormone

To illustrate the complexity of the experiments, let's look at the first experiment in this module, "Metabolism". In short, the student has at his disposal a virtual mechanical apparatus for measuring oxygen consumption, the mode of operation of which is explained in the text preceding the experiment. Also, there are three experimental animals available, one of which is a control animal, the other has a removed thyroid gland and the last one has had its pituitary gland removed. Finally, three treatments are available (propylthiouracil, thyrostimulating hormone, and thyroxine). Using all the above, based on the theoretical knowledge summarized in the supplementary material, the student has the task to measure the oxygen consumption of the experimental animal, explain the role of the thyroid gland and its hormones in maintaining metabolic rate, and to assume the action of negative feedback as one of the basic control mechanisms in the work of endocrine systems. We see that it is necessary for the student to prepare in advance for a given experimental exercise and to know the physiological principles of the functioning of one endocrine axis (pituitary-thyroid gland). One must also notice the connection between thyroid hormones and the rate of metabolism. Furthermore, it must make an assumption about

how certain changes in the work of the parts of the said axis will affect the metabolism of a specific test animal. Finally, his goal is to test this assumption by choosing the right experimental animal and/or treatment and to draw a conclusion based on the results of the experiment. All this requires higher cognitive processes and strengthens previously acquired theoretical knowledge. Of course, the described experiment could be performed in a real laboratory, which would give the student experience in working with experimental animals, as well as in performing surgical operations that are necessary in the described case. However, that experiment would require a laboratory extremely well equipped for surgical interventions (considering the mentioned removal of the pituitary gland of one of the rats), an experienced surgeon who would train students for those interventions, a whole team of assistants, not to mention a relatively large number of experimental animals. Namely, we could not use only three animals for the described experiment (as shown in the PhysioEx software package), due to their individual differences and the necessary statistical analyzes that would necessarily accompany the experiment. If we were to have experimental groups of minimal numbers, we would have to use at least 72 animals. In the end, it would take a long time.
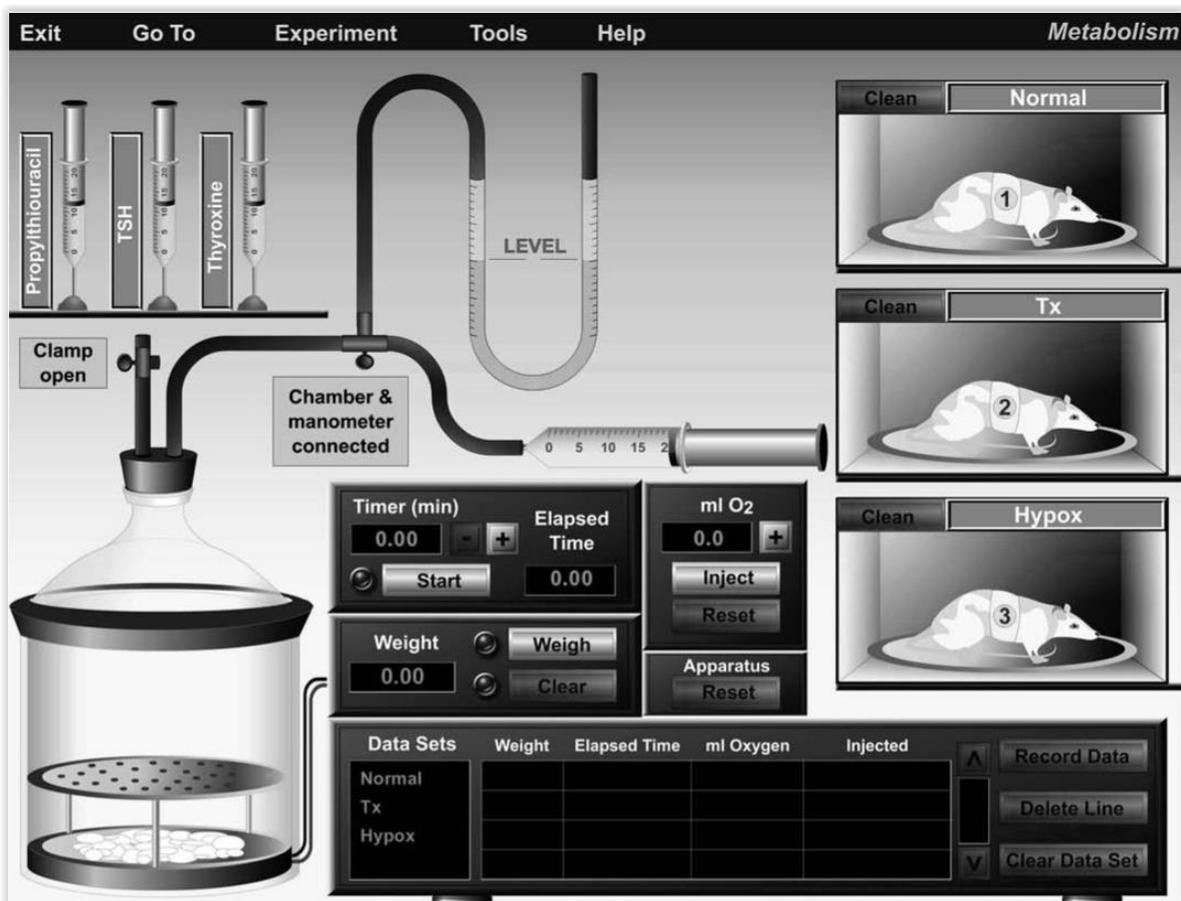
Figure2. Determining the rate of metabolism - setting up an experiment. Downloaded from:
https://media.pearsoncmg.com/bc/bc_physioex_8/sheets_ap/worksheet_ap04.pdf

As the connection between the described experimental groups, treatments and metabolism rate is well known, it was possible to design a program simulation of the experiment that emphasizes the results of real experiments well enough. These results are reached quickly and the conclusions are reliable and direct the student to the correct conclusions. All this makes the described software an ideal solution for supplementing biology teaching. Moreover, additional material in the form of a table in

which students enter the results of the experiment and a short questionnaire leads the student to the set goal and teaches them a systematic approach to work in the laboratory.

During the previous ten years, it has been shown that the work in the described program package is interesting for students and that it greatly contributes to the achievement of the goals of teaching physiology/endocrinology. Moreover, we tested the use of this software in working with high school students and the results were very encouraging. The students quickly, actively and independently overcame the set problems, with the guidance provided by the teacher. Thus, we came to the conclusion that the use of PhysioEx software package is very useful in the teaching we conduct and stimulates us to look for other solutions in the field of information technology and thus achieve an even more proactive approach to meeting the set teaching goals.

CHART 1 — Effects of Hormones on Metabolic Rate	Normal rat	Thyroidectomized rat	Hypophysectomized rat
Baseline			
Weight	_____ grams	_____ grams	_____ grams
ml O_2 used in 1 minute	_____ ml	_____ ml	_____ ml
ml O_2 used per hour	_____ ml	_____ ml	_____ ml
Metabolic rate	_____ ml O_2/kg/hr	_____ ml O_2/kg/hr	_____ ml O_2/kg/hr
With thyroxine			
Weight	_____ grams	_____ grams	_____ grams
ml O_2 used in 1 minute	_____ ml	_____ ml	_____ ml
ml O_2 used per hour	_____ ml	_____ ml	_____ ml
Metabolic rate	_____ ml O_2/kg/hr	_____ ml O_2/kg/hr	_____ ml O_2/kg/hr
With TSH			
Weight	_____ grams	_____ grams	_____ grams
ml O_2 used in 1 minute	_____ ml	_____ ml	_____ ml
ml O_2 used per hour	_____ ml	_____ ml	_____ ml
Metabolic rate	_____ ml O_2/kg/hr	_____ ml O_2/kg/hr	_____ ml O_2/kg/hr
With propylthiouracil			
Weight	_____ grams	_____ grams	_____ grams
ml O_2 used in 1 minute	_____ ml	_____ ml	_____ ml
ml O_2 used per hour	_____ ml	_____ ml	_____ ml
Metabolic rate	_____ ml O_2/kg/hr	_____ ml O_2/kg/hr	_____ ml O_2/kg/hr

Figure 3. Determination of metabolism rate - presentation of results. Downloaded from:
https://media.pearsoncmg.com/bc/bc_physioex_8/sheets_ap/worksheet_ap04.pdf

Review of the TeComp project contribution on teaching modernization

In addition to the previously described interactive virtual solutions that are already used in the teaching process at the Faculty of Biology - University of Belgrade, a great contribution to the modernization of teaching has enabled participation in the project from the Erasmus+ program, "Strengthening Teaching Competences in Higher Education in Natural and Mathematical Sciences (TeComp)", whose co-financing was provided by the EACEA. Namely, during the project, so far we have been able to hear how colleagues from higher education institutions from Spain, Slovakia, the Czech Republic, Belgium and Albania approach this problem. Among the offered solutions, there are many that find application in biology teaching, and some of them are already used in regular teaching at the Faculty of Biology, University of Belgrade. This especially refers to the knowledge that we have had the opportunity to acquire so far by participating in the workshop "Professional Development in Educational Interaction and Communication", organized and conducted by the University of Ghent. As a result, in the courses of the Faculty of Biology, University of Belgrade, in which the authors of this text are directly involved, recommendations adopted regarding topics such as posters, discussions and collaborations, animation are already used in regular classes. Student reactions are positive and progress were made in meeting teaching objectives. Thus, the benefit of participating in the TeComp project has already been achieved and one of its goals has been fulfilled, ie. the quality of biology teaching at the Faculty of Biology, University of Belgrade has been improved. This is a good start and it is necessary to share the acquired knowledge and skills with colleagues who are willing to invest personal effort and time to modernize their approach to teaching and make it more interactive, interesting and methodologically richer.

References

PhysioEX Exercise 4, Endocrine System Physiology [Online]. Pearson. Available: https://media.pearsoncmg.com/bc/bc_physioex_8/sheets_ap/worksheet_ap04.pdf [Accessed 27.4. 2021].

AKINOGLU, O. & TANDOGAN, R. Ö. 2007. The Effects of Problem-Based Active Learning in Science Education on Students' Academic Achievement, Attitude and Concept Learning. *Eurasia Journal of Mathematics, Science & Technology Education,* 3**,** 71-81.

ASYARI, M., AL MUHDHAR, M. H. I. & IBROHIM, S. H. 2016. Improving critical thinking skills through the integration of problem based learning and group investigation. *International Journal for Lesson and Learning Studies,* 5**,** 36-44.

BRUNDTLAND, G. H. 1987. United Nations. Report of the World Commission on Environment and Development: Our Common Future.

DORTEL, E., PECQUERIE, L. & CHASSOT, E. 2020. A Dynamic Energy Budget simulation approach to investigate the eco-physiological factors behind the two-stanza growth of yellowfin tuna (Thunnus albacares). *Ecological Modelling,* 473**,** 109297.

DRIVER, R., NEWTON, P. & OSBORNE, J. 2000. Establishing the norms of scientific argumentation in classrooms. *Science Education,* 84**,** 287-312.

GRANT, R. 1997. A claim for the case method in the teaching of geography. *Journal of Geography in Higher Education,* 21**,** 171-185.

JERONEN, E., PALMBERG, I. & YLI-PANULA, E. 2017. Teaching Methods in Biology Education and Sustainability Education Including Outdoor Education for Promoting Sustainability—A Literature Review. *Education sciences,* 7.

KARAMI, M., PAKMEHR, H. & AGHILI, A. 2012. Another view to importance of teaching methods in curriculum: collaborative learning and students' critical thinking disposition. *Procedia Social and Behavioral Sciences,* 46**,** 3266-3270.

KATZ, S., HOLLINGSWORTH, R. G., BLACKBURN, J. G. & CARTER JR, H. T. 1978. Use of computer simulation in teaching physiology: Student laboratory. *Computers & Education,* 2**,** 307-318.

KERN, E. & CARPENTER, J. 1986. Effect of field activities on student learning. *The Journal of General Education,* 34**,** 180–183.

KESELMAN, A. 2003. Supporting inquiry learning by promoting normative understanding of multivariable causality. *Journal of Research in Science Teaching,* 40**,** 898–921.

PRINCE, M. J. & FELDER, R. M. 2006. Inductive teaching and learning methods: Definitions, comparisons and research basis. *Journal of Engineering Education,* 95**,** 123-138.

REHAN, R., AHMED, K., KHAN, H. & REHMAN, R. 2016. A way forward for teaching and learning of Physiology: Students' perception of the effectiveness of teaching methodologies. *Pak J Med Sci,* 32**,** 1468-1473.

SANTHAKUMARI NAGOTHU, R., YOGANANDA REDDY INDLA, Y. & PALURU, R. 2016. Effective physiology teaching methods: from the perspective of first year MBBS students. *Indian J Clin Anat Physiol,* 3**,** 336-338.

SIMMONS, M., WU, X., KNIGHT, S. & LOPEZ, R. 2008. Assessing the influence of field- and GIS-based inquiry on student attitude and conceptual knowledge in an undergraduate ecology lab. *CBE—Life Sciences Education,* 7**,** 338-345.

SRI NAGESWARI, K., MALHOTRA, A., KAPOOR, N. & KAUR, G. 2004. Pedagogical effectiveness of innovative teaching methods initiated at the Department of Physiology, Government Medical College, Chandigarh. *Adv Physiol Educ,* 28**,** 51-58.

VESELINOVSKA, S., GUDEVA, L. & DJOKIC, M. 2011. Applying appropriates methods for teaching cell biology. *Procedia Social and Behavioral Sciences***,** 2837-2842.

VUJOVIC, P. 2016. Improving teaching skills: from interactive classroom to applicable knowledge. *Adv Physiol Educ,* 40**,** 1-4.

ZAO, P., STABLER, T. & PETERSON, G. 2005. PhysioEx 5.0: Laboratory Simulations In Physiology CD-ROM Version. Pearson.

Authors:
Tatjana Anđelković
Ivana Kostić

QUESTIONS:

What makes a remote access session?

How to connect instruments and make them available for remote access?

What are the technical prerequisites for applying remote approach?

How to prepare a remote access session adequately pedagogically and methodically?

How to use NETCHEM platform for remote access to instruments?

Methodological scenario: Remote access to the laboratory with instrument control (ion chromatograph).

Remote Access to Analytical Instruments in Chemistry Higher Education – From Idea to Realization

Content

Introduction

Guidelines for the application of remote access to analytical instruments in the field of chemistry in higher education, presented in this material, were developed during the implementation of the Erasmus+ project „ICT Networking for Overcoming Technical and Social Barriers in Instrumental Analytical Chemistry Education" - NETCHEM (2016-2020).

The aim of this material is to present to the user of this teaching and learning technique (individual or institution) some of the most important determinants of the organization of an effective class in which remote access to analytical instruments is used as a teaching method.

The guidelines set out should not be sensed as the best and the only approach to this type of learning/ teaching. The idea is to point out the possible problems, shortcomings and advantages of this type of teaching by setting these guidelines, with the suggestion that it is always important that the user of this teaching adapts to his needs and technical capacities the way he will implement this type of teaching. Therefore, these guidelines should be considered more in the concept of setting a general strategy for the implementation of remote access to analytical instruments within inter- or intra-institutional teaching. Of course, the possibility of access to developed NETCHEM platform for this purpose of learning/teaching is offered (and recommended) as one of the possible options for implementing remote access in chemistry teaching.

What makes a remote access session session?

Remotely accessible analytical instruments allow students, teachers and researchers to access the instrument and control the analysis they perform on these instruments from another remote location, from another laboratory, from the classroom or even from home. In this way, the barrier that exists due to the impossibility of physical access of students and teachers to the instrument is reduced, with the aim of performing an authentic and realistic educational activity.

Since almost all modern chemical laboratories of instrumental analysis today use instruments that are controlled via a computer interface, remote control of instrumentation is not only relatively easy to perform, but raises the level of education of students in understanding the principles and mastering the technique of working with instruments. real research and problem situation.

Laboratory exercises and experiments form an essential part of any study program in chemistry, biology and physics. For the realization of this practical part of teaching, the possibility of using instruments and apparatus directly by students is important. The application of "distance work" in the laboratory part of the courses in chemistry, biology and physics enables access to experiments on instruments and apparatus that the institution does not have at the moment. Also, in this way, students, teachers, researchers can work on more complex and expensive instruments that are often controlled by specialized and licensed software and drivers, and which cannot be easily installed on the remote side of the user.

Taking into account the fact that there could be many different scenarios in achieving remote access to analytical instruments, it is necessary to define terms in clarifying this area.

Any communication that takes place between the laboratory with the analytical instrument on the one hand and the user (educator, student, researcher) on the other hand via internet interaction and devices (computer, laptop, tablet and mobile phone) is called a **remote access session** to the analytical instrument.

Two parties are participating in the session:
- a **session host** that is defined as a computer that invites other users to join the session and
- a **session guest** page defined as the PC that joins the session invitation.

Also the remote access session of the analytical instrument includes two key sides:
- a **remote access laboratory** (where the analytical instrument is located),
- a **remote access classroom** (a place where a user who remotely accesses the instrument works).

A remote access session to an analytical instrument represents a step beyond computer-generated laboratory simulations. They represent an alternative to working in a real laboratory. Remote teaching laboratories can be used in four ways:

1. to demonstrate and observe the experiment;
2. for conducting measurements (especially in real time);
3. for manipulating instruments in experiments;
4. for remote cooperation.

Demonstration, ie observation of a remote experiment, is by far the simplest and most powerful version of a remote experiment. Usually, the interaction of the observer with the "remote lecturer" is minimal and is often limited to the control of, for example, cameras (control of an astronomical camera or electron microscope).

Measurement. Examples of direct measurement and monitoring of real-time results are measurement of reaction kinetics in chemistry, thermal conductivity experiments, chemical analysis by gas / liquid chromatography, or crystal crystal diffraction measurement.

Manipulation. In addition to observation and measurement, some remote experiments require real physical control. This could be electric motor control or more interactive operations, such as adjusting some analysis parameters and the instrument.

Cooperation. Certainly, one of the advantages of the Internet is that it can make it easier to share not only experiments and instruments, but also data. In this way, the interactive cooperation of observers and lecturers provides a better realization of the experiment.

How to connect instruments and make them available for remote access?

Remote access offers an actual measurement experience. Instruments used in remote access are assumed to have a LAN (local area network) or WAN (wide area network) connection. For the realization of such teaching activity, prior adequate organization and preparation of classes is required, which implies the engagement of not only teachers and students, but also technicians in charge of working on the instrument, as well as IT experts. Precisely due to the complexity of the organization of this teaching, as well as the need to know how to work on instruments that are often complex and sophisticated, this teaching method is more often used in practice within higher education or lifelong learning, or continuing professional development of experts in chemical detection and sample analysis. The fact that this method of work, in addition to its application in the research aspect, has application in teaching, shows the need to introduce an appropriate pedagogical view on the application of remote access.

Examples of instruments that can be successfully applied in remote access sessions are instruments that are otherwise available on the NETCHEM platform, which provides the ability to connect to instruments within the following instrumental techniques: UV-VIS spectrophotometry, Atomic absorption spectrometry, Gas and Liquid chromatography , Mass spectrometry, Thermo-gravimetric analysis. For example, online interpretation of chromatograms, spectral data, identification of compounds by mass spectrometry, databases, and fragmentation patterns are just a few examples in which a chemistry student can benefit greatly.

What are the technical prerequisites for applying remote approach?

Some basic characteristics in terms of technical requirements for a successful remote access session are:

- istrumental analytical equipment,
- audio / video equipment installed in the laboratory,
- audio / video equipment installed in the classroom,
- software requirements,
- connections and signals.

The basic elements to be provided are: computers with access to install add-ons and software, high-speed Internet access, speakers, microphones, video projector (for multiple viewers), Internet voice protocol software with username and password (preferably Skype), Team Viever. Prerequisite for successful implementation of these guidelines is knowledge of using TeamViewer and / or Skype for business

An example of the necessary equipment used for distance learning in the curriculum of chemistry is according to the plan of the NETCHEM project:

–Laboratory equipment for remote access

- Analytical instrument available for remote access,
- TeamViewer and / or Skype for business software,
- Lan Network: routers (Router 10 Gbs fiber optics + 25 PORT LAN Allied Telesis AT-GS924MX-50, Gigabit Ethernet Managed switch with 24 ports 10/100 / 1000T Mbps, 2 SFP / Copper combo ports, 2 SFP / SFP + uplink slots , single fixed AC power supply; WI-FI Router 802.11 ac standard, D-Link DIR-809, AC750 Dual-Band,
- Camera Stabilizer (DJI Osmo Mobile 3-Axis Handheld Stabilizer for Phone Camera (drivers for Android & iPhone),
- Spotlight with LED lighting (max. Height 200 cm, two LED reflectors).

–Classroom equipment for remote access

- PC Monitor (Display diagonal 21.5 - 22 "Res. FULL HD 1920×1080, ports: HDMI / DVI, with HDMI / DVI cable),
- PC desktop (Intel CPU LGA1151 Intel® Celeron® Dual-Core G3900, 2.8GHz BOX 14nm; AMD 'APU FM2 AMD A6-6400K, 3.9GHz / Radeon ™ HD 8470D or better GIGABYTE MB - SOCKET 1151 - GA-H110N, 8GB RAM, 256 GB SSD, USB 3.0, LAN, Audio, USB 2.0, HDMI & DVI, mini ITX case, $56 \times 220 \times 200$ mm,
- Multi-device keyboard and mouse (Wireless Bluetooth) (Keyboard LOGITECH K780, K480, K380, Mouse LOGITECH Thriatlon),
- PC headset (WIRED: MICROSOFT LifeChat LX-3000, USB 2.0; WIRELESS: Bluetooth connection, on-ear or over-ear, with microphone, foldable, Near Field Communication (NFC), e.g. SONY MDR-ZX220BTB, Bluetooth black),
- WEB cameras (720p - Logitech C270; HD 720p middle class, USB compatible with Skype and Win 10; 1080p Full HD - Logitech C922 Pro stream USB, HD Web camera higher class Full HD 1080p at 30 fps & 720p at 60 fps st compatible with Skype and Win 10),
- Convertible laptop (2-in-1) Microsoft Surface Pro 12.3 i5 8Gb 256GB Multi-Touch Tablet 2017 Model (12.3 "10-Point Touch Display, 2736×1824 Screen Resolution, Windows 10 pro,

USB 3.0, Mini DisplayPort, microSD Slot , 2.6 GHz Intel Core i5-7300U (Dual-Core) Weight <1 kg, Keyboard + pen; Var 1: 8 GB, 256 GB SSD; Var 2: 4 GB, 128 GB SSD),

– 55 "LCD TV (Display diagonal 55" Full HD 1920 × 1080, USB, HDMI),

– Screen for Video beam - projector (Diagonal min. 250 cm),

– Wireless display Adapter MIRACAST (Microsoft) Wi-Fi CERTIFIED™ Miracast® technology, USB / HDMI, remote connection with a 23-foot range or more, Adapter length: 103.5 × 22 × 11 mm, USB extension cable: 159 mm

– Presentation pointer (Logitech Spotlight presentation remote MS PowerPoint, Win 10; Spotlight connects instantly via USB receiver or Bluetooth®. It's plug-and-play on most platforms and compatible with all popular presentation apps. Plus, it's fully rechargeable and boasts a 100 -foot operating range).

How to adequately pedagogicaly and methodologicaly prepare a remote access session?

Here are some basic features to consider when defining a remote access session scenario type: A. What is the purpose of remote access?

a. Educational - one side trains the other,

b. Consultative or research - both parties have comparable skills and knowledge,

c. The far side.

B. What are the main characteristics of a learning group:

a. What is the level of knowledge of the group?

b. What is the size of the group?

c. What is the number of groups (in parallel)?

C. What is the type of information exchange?

a. The group is predominantly receptive,

b. The group is predominantly interactive.

D. What kind of interactivity is achieved?

a. Single monitor / software interaction

(A) with -Tim instrument only To the viewer, the lecture is face to face;

(B) or just Skype - no interaction with instrument (audio video connection between participants only)

b. Dual interaction of monitor and software (with instrument and web conference)

To provide a better classification of possible remote access scenarios, Table 1 summarizes some basic characteristics of remote access sessions.

There are many different pedagogical / methodological scenarios in which remote access can be achieved. For example, one of the scenarios is a demonstration of the work of an analytical instrument via real-time video conferencing software, where communication is realized between analyst-educators and students who are at a remote location from the instrument. Either a scenario in which the student can have remote control over the instrument or a scenario that provides for a

demonstration of the instrument but also remote control by the student, or a scenario in which only the instrument or its parts are shown. Also, the realization of teaching can vary significantly depending on what IT equipment is used, cameras, headphones, microphone or video conferencing option.

Also, the teacher can choose to send his samples for analysis to one of the remote access institutions in order to analyze those samples, or he can use the samples offered by the host institution (the institution where the analytical instrument is physically located). In order to successfully implement such a class or in a broader sense remote access sessions, it is necessary to consider the following steps: contact the host institution to agree on the type of instrument; state the day and time of the test session and the final session, which would include students; send your sample for analysis or confirm the use of a sample proposed by the host institution; ensure the presence of educators for the implementation and conduct of classes and technicians who are in the host institution and who perform the chemical analysis of the instrument.

Table 1. Characteristics of a remote access session to an analytical instrument

Communication sides		
host side	participant's/PC of instrument in laboratory/office	
guest side	participant's/PC of instrument in laboratory/office	
Communication software		
Team Viewer	Meeting:	Yes/No
	Remote control:	Yes/No
	Meeting and Remote control simultaneously:	Yes/No
Skype	Call 1:1	Yes/No
	Conference Call	Yes/No
Communication hardware		
on host side		
on guest side		
Information exchange type:		
1. Educational (one side is dominantly receptive):	Yes/No	
Place of Educator participant:	host side/guest side	
Number of educator(s):		
Place of student participant:	host side/guest side	
Number of student participant(s):		
2. Consultative (two sides are equal in giving-receiving information):	Yes/No	
Number of host side participant(s):		
Number of guest side participant(s):		

How to use netchem platform for remote access to instruments?

1. Open NETCHEM website www.netchem.ni.ac.rs and chose **Platform** option in the Main menu (figure 1). Also, you can open NETCHEM platform direct using web address http://netchem-eu.com/home.html

2. After opening the platform, click on **Remote Access** option in the Main menu (figure 2).

3. Choosing this option give you possibility to see which instruments are offered by partner institutions (figure 3). Also, on this page, there are options for available Laboratory Guides and other Education materials, as well as option for Remote Session Request. Also, there are names and e-mail addresses of contact persons for remote access at institution. For the example, available instruments at University of Niš are presented in the following picture (figure 4).

4. When you chose instrument, you would like to use during remote access, you have to fill in the Remote Session Request formular. After click on Remote Session Request option, Remote Access Request form will be opened (figure 5). The first part in filling in Remote Access Request form is part that contains your personal information.

5. The second part requires details about Remote Session. It includes choosing of Partner institution, Type of your institution and approximately how many participants will participate remote session (figure 6). Also, you have to fill in dates and times for first and second test dates, as well as date and time for main remote session. At the end, you have to chose option about how much time requested session requires.

6. The third part of this formular requires some background information such as science area, science level of participants, type of software that will be used for remote access (TeamViewer or Skype) and type of samples (figure 7).

7. After filling in the formular, input of code from the image is needed.

8. If you filled in all required fields correctly, the last step is click on Submit button.

9. After completing all steps you will get the e-mail with all details that you are put in the formular and contact person who are responsible for required instrument will contact you by e-mail. All further communication will be realised by e-mail.
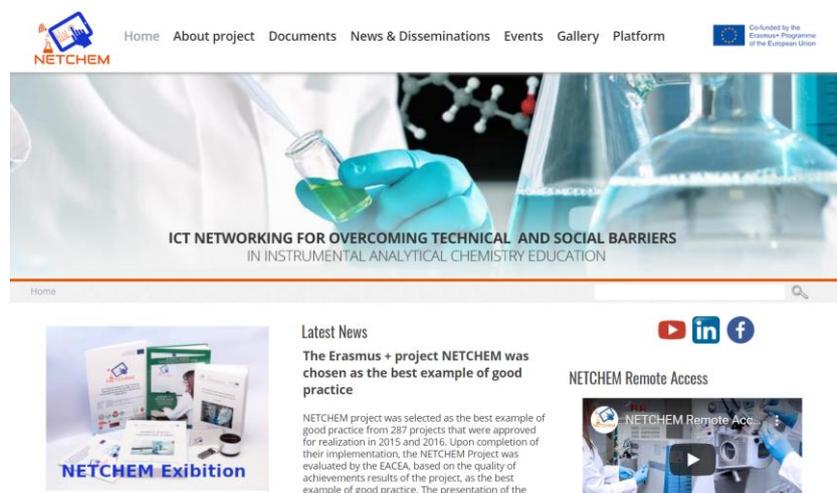
Figure 1. Platform option in the Main menu on www.netchem.ni.ac.rs
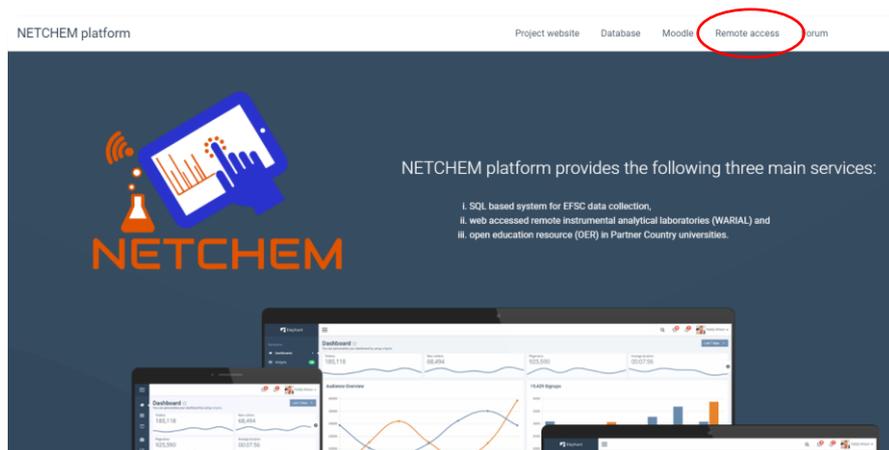
Figure 2. **Remote Access** option in the Main menu

Remotely Accessible Analytical Instruments

Remotely Accessible Analytical Instruments allows researchers and students to access and control their analysis out of their laboratory, in the classroom, or even at home.

NETCHEM network of remotely accessible analytical instruments lowers the barrier-to-entry and enables instructors to deliver authentic educational activities for students.

Please select a center to obtain information about available instruments in the center.

- **Partner 1** – University of Niš
- **Partner 2** – Universite Pierre et Marie Curie
- **Partner 3** – University of Greenwich
- **Partner 4** – Brno University of Technology
- **Partner 5** – CEA
- **Partner 6** – University of Belgrade
- **Partner 7** – University of Novi Sad
- **Partner 8** – University of Kragujevac
- **Partner 9** – Agricultural University of Tirana
- **Partner 10** – University of Tirana
- **Partner 11** – Analysis d.o.o.
- **Partner 12** – Enološka stanica Vršac
- **Partner 13** – Zlatiborac
- **Partner 14** – AQualEER

Remote Access Laboratory Guides

Remote Session Request

Education material usage during academic year 2018/2019

Figure 3. Instruments are offered by partner institutions

Partner 1 – University of Niš

Analytical Instruments available for remote access:

- GC-MS
- ESI-MS Decca
- ESI-MS Advantage
- HPLC + ESI-MS LTQ Orbitrap
- UV-VIS
- AAS
- Dionex Aquion Ion Chromatograph
- Stereo Microscope Leica MZ-16A with camera
- Light Microscope Leica DM 1000 with camera
- Telescope Meade LX200-ACF 8"
- Lunt LS60THA Solar Telescope

Contact for remote session: Darko Anđelković
e-mail: darko.andjel@outlook.com

Contact for remote session: Tatjana Anđelković
e-mail: tatjana.andjelkovic@outlook.com

GC-MS (Gas chromatography – mass spectrometry)

- GC: Hewlett Packard 6890 series GC System with autosampler
- MSD: Agilent 5973 Mass Selective Detector (Electron Ionization MSD-EI, single quadrupole)
- Column: HP DB5 non-polar & several others capillary columns

GC-MS is the analysis method of choice for smaller and volatile molecules such as benzenes, alcohols and aromatics, and simple molecules such as steroids, fatty acids, and hormones. It can also be applied towards the study of liquid, gaseous and solid samples.

Figure 4. Available instruments at University of Niš

Figure 5. Remote Access Request form

Figure 6. Choosing of Partner institution, Type of your institution and approximately how many participants will participate remote session

Figure 7. Science area, science level of participants, type of software that will be used for remote access (TeamViewer or Skype) and type of samples

Methodological scenario: remote access to the laboratory with instrument control (ion chromatograph)

The application of this methodological scenario is suitable in cases when it is necessary to present operation of an instrument, the use of software, instrument setup or sample analysis to a larger group of students (more than 5).

The advantage of this type of approach is reflected in the fact that a large group of students is able to follow working and adjustment of the instrument, which is not possible in laboratory conditions, regardless of the size of the laboratory. Also, taking into account the size of the computer monitor, in laboratory conditions, a maximum of 5 students can simultaneously and equally follow the settings on the monitor. Using this methodological scenario, students are in the same room and follow the settings through a projector on the screen and all students have an equal opportunity to see the settings and the operation of the instrument, as well as the interpretation of the results. In laboratory conditions, even when dividing students into smaller groups, it is not possible for all students to be presented with lectures or exercises in an identical way, which is possible by applying this type of methodological scenario.

The disadvantage of this type of methodological scenario can be reflected in the possibility of problems in internet communication between computers in the classroom, or any room where the target group is located, and computers in the laboratory, which is connected to the instrument. Also, with instruments that require manual entry of the sample, i.e. does not have an autosampler, it is necessary for another person to participate in the teaching and be present in the laboratory.

–Procedure for realization of remote access to the instrument

For the realization of this type of methodological scenario, it is necessary to have certain equipment, software and internet communication.

Necessary equipment

Equipment required to perform remote access to the instrument:

– Computer connected to the instrument which will be adjusted and controled,

– Computer in the classroom or amphitheater (location of students)

– Projector

– Video beam

Software necessary for the realization of the methodological scenario:

– TeamViewer program

It is necessary for the TeamViewer program to be installed on the both computers (computer located in the laboratory and connected to the instrument and the computer located in the classroom where the target group of students is and which will be used for presenting of instrument working and interpretation of the results).

In addition to the listed equipment and software, it is necessary to have an internet connection between the computer in the laboratory and the classroom where the lecturer and students are.

Beginning of communicaton

The target group are students who are in the classroom or amphitheater together with the lecturer.

The instrument is located in the laboratory, and depending on the type of instrument and the method of sample analysis or adjustment, it may be necessary for another person to be present in the laboratory.

If the lecturer already has a computer connected to TeamViewer in his list that he wants to access, it is not necessary for an assistant to be present in the laboratory.

If the lecturer does not have in his list within TeamViewer a computer connected to which he wants to access, it is necessary for the assistant to be present in the laboratory and to report the parameters for accessing the computer via a certain connection (phone, skype, etc.). has requested information (parameters).

Communication

The lecturer accesses the TeamViewer program on the computer. If he has his own account, he connects directly to the computer in the laboratory.

If he does not have his account or does not have the computer within his list, contact the assistant in the laboratory for the necessary information (ID and password) which allows access (Figure 8. Access to TeamViewer©-u.
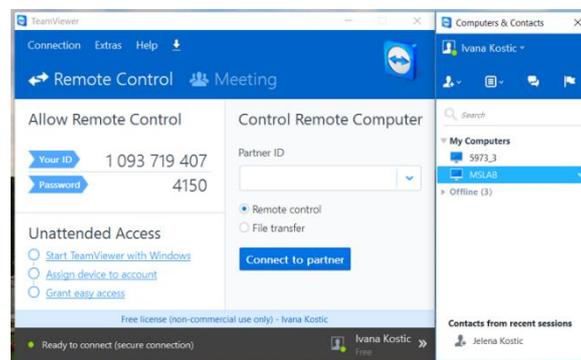
Figure 8. Access to TeamViewer©-u

Also, there is a possibility for the lecturer to obtain the required parameters in advance and than immediately access the computer connected to the instrument.

Upon successful access, the same veiw of the computer monitor connected to the instrument will appear on the computer monitor located in the classroom. (Figure 9.
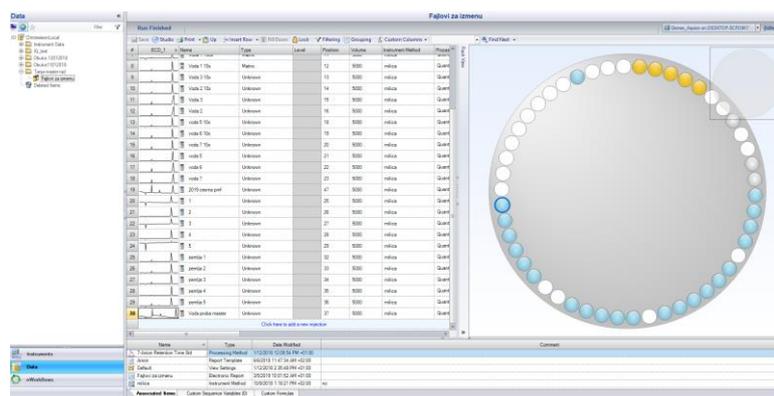
Figure 9. PC monitor for control of IC

After accessing the instrument, all students in the room can follow the settings and review the obtained analysis results in the same time.

−Outcome of the applied methodological scenario

After the class the main outcome is that all students in the room were equally able to follow the process of settings and analysis of the sample on the instrument.

This methodological scenario is extremely suitable for performing demonstrative exercises, when it is necessary for students to learn certain practical skills of handling a certain instrument.

References

1. Guidelines for courses publishing by WARIAL networking, http://www.netchem.ac.rs/docu-ments?task=download.send&id=1701&catid=324&m=0

2. Linda R. Phipps, Creating and Teaching a Web-Based, University-Level Introductory Chemistry Course That Incorporates Laboratory Exercises and Active Learning Pedagogies, J. Chem. Educ. 2013, 90, 568−573

3. Baran, J.; Currie, R.; Kennepohl, D. Remote Instrumentation for the Teaching Laboratory. J. Chem. Educ. 2004, 81, 1814−1816.

4. Dietmar Kennepohl and Lawton Shaw, Accessible Elements Teaching Science Online and at a Distance, AU Press, Athabasca University, 2010.

Author:
Goran Radojev

QUESTIONS:

How to visualize various mathematical problems with the help of GeoGebra?

How to write programs in the Wolfram Mathematica software package that will animate more difficult mathematical problems?

With GeoGebra and Wolfram Mathematica, can students more easily understand and process more complex mathematical concepts and procedures?
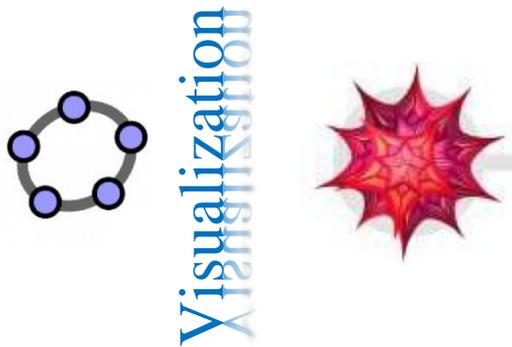
Visualization problems using GeoGebra and Wolfram Mathematica

Contents

Introduction

This text is intended to help teachers who want to use software GeoGebra and Wolfram Mathematica, in order to improve the teaching process, whether it is classical or on-line teaching. The students often have the problem that different definitions, mathematical problems and models cannot be easily visualized and un- derstood, so the dynamic environment provided by these two software is a great support in such situations. In Serbia, several PhD thesis have been written on this topic, see [2, 3].

In order to be able to follow and understand this text, it is necessary to have some previous work experience with these two mentioned software. Unless, of course, you have not been able to get acquainted with software GeoGebra and/or Wolfram Mathematica, it would be desirable to get acquainted with those first, using of appropriate literature. For this purpose, we suggest: [4] – for software GeoGebra i [1] – for software Wolfram Mathematica.

The manuscript is divided into three parts. In the first chapter we will get acquainted with some important GeoGebra tools, which allow you to create dynamic applets. In the second chapter you will get to know the commands in Wolfram Mathematica, with which we will create animations. Finally, in the last chapter, we will present three problems, in both software, that will allow students easier visualization and understanding the given problems.

After reading this text, we believe that you will be able to create various applets and animations, which you can use it during classes, but also students during independent work.

This manuscript was created as a result of work on the Erasmus+ project TeComp.

GeoGebra

GeoGebra is dynamic mathematical software in which you can display and solve problems in geometry, algebra, analysis, statistics, ... This software was developed by Markus Hohenwarter and an international team of developers. Name of this software is composed of the words GEOmetry and alGEBRA, [2]. It is completely free to download and use of this software, so that it is easily accessible to everyone. There is also a mobile version of application GeoGebra for smartphones, which support both Android and iOS. So, in order for students to use GeoGebra, they do not even need to have a computer in front of them, it is just enough to have a mobile phone, which makes it even easier to use. GeoGebra is translated in dozens of different languages.

In this chapter, we will deal, first of all, with the options shown in Figure 1.1.
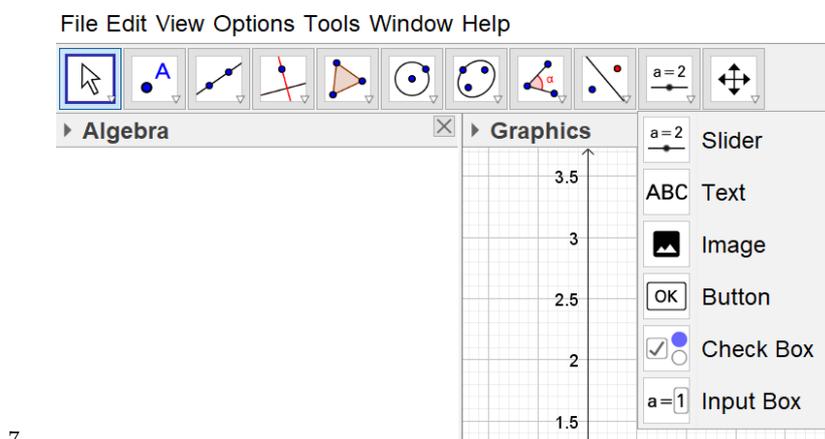
7

Figure 1.1: GeoGebra – interactive window.

So, we pay special attention to the following options:

- Slider;
- Check Box;
- Input Box.

8
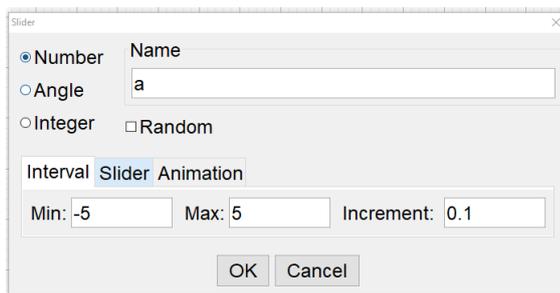
9

Figure 1.2: Using the slider.

10

The option "Slider" allows us to create various animations that make it easier for us to understand various problems. Let's show an example of drawing a graphic of the function $y = 2^x$. When you ask students for the definition of the term graphics of the function - you will get various answers, but rarely which will be completely accurate. More often, the graphics of the function f is associated with

a (continuous) curve, which is basically wrong. There is a small number of those who say that it is a set of points $\text{Gr}_f = \{(x, y)|y = f(x)\}$ GeoGebra allows us to show this through the use of a slider. If we set the slider, a dialogue box will appear, see Figure 1.2.

It is necessary to enter the interval at which the slider will move, with a defined step. Now if in the input field we define point $A(a, 2^a)$, by changing the value on the slider, the position of this point will also change. To see how the graph of this function looks like, we will include a trace of this point as in the Figure 1.3.

11
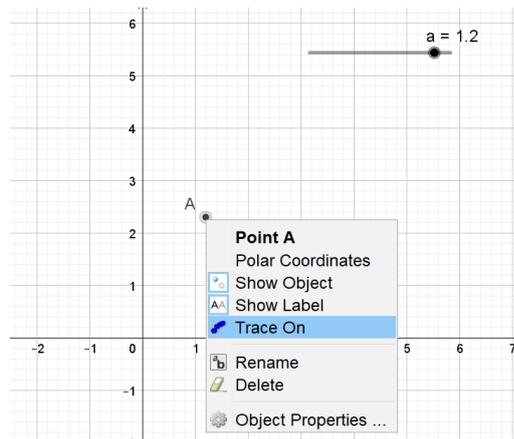
12

Figure 1.3: Trace.

Now, by changing the value of the slider on the interval which is defined, point A will describe the set of points which are located on the graphics of the function $y = 2^x$ (Figure 1.4) and thus it is easier to adopt the concept of the graphics of the function through visualization.

13
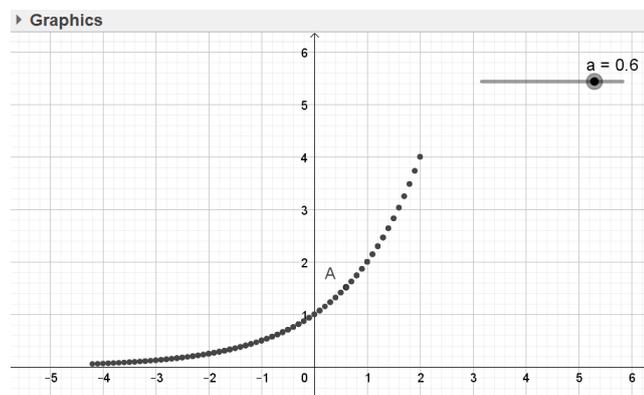
Figure 1.4: Points of the graphics of the function $y = 2^x$.

 Let's add the function $y = 2^x$ (in another color) to show how we can use the "Check Box" tool. Selecting this option opens a window in which you need to connect this field with some of the objects. In our case, this is the function f (Figure 1.5). In this way we get a field, which shows the function, if it is checked. Otherwise, this function will not be displayed in GeoGebra, Figure 1.6.
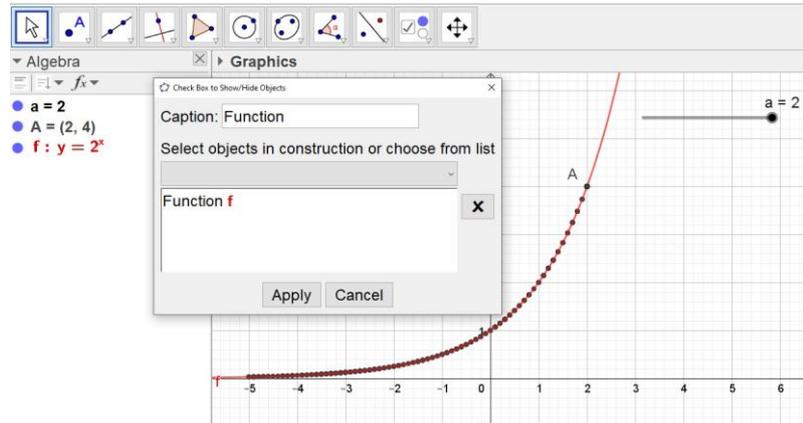
Figure 1.5: Check Box.
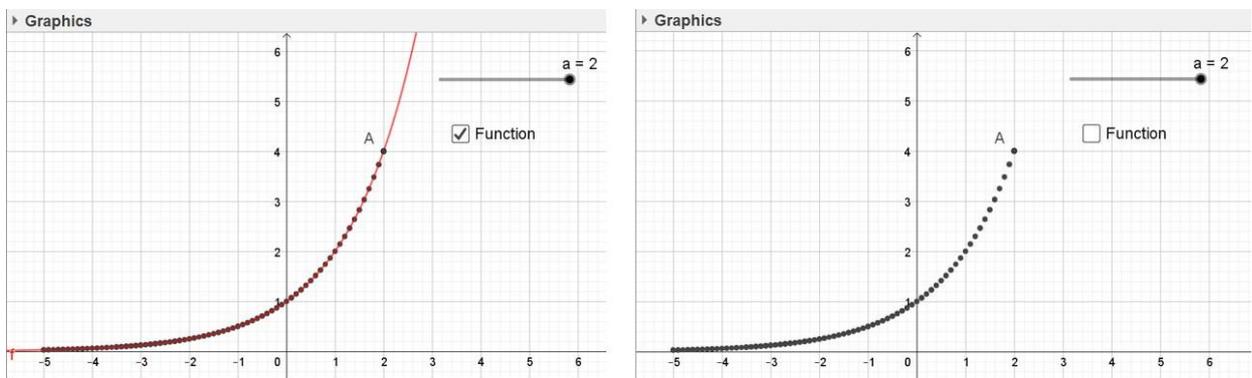
Figure 1.6: Checked and unchecked field.

Let's also show the option "Input Box". Before that, we define another function, for example $g(x) = 3^x$ which we want to draw on the same figure and compare with the function f. If we choose the mentioned option, dialog box will be displayed as in Figure 1.7. It is necessary to connect this field with some of already defined objects – in our case, it will be a function g.

15
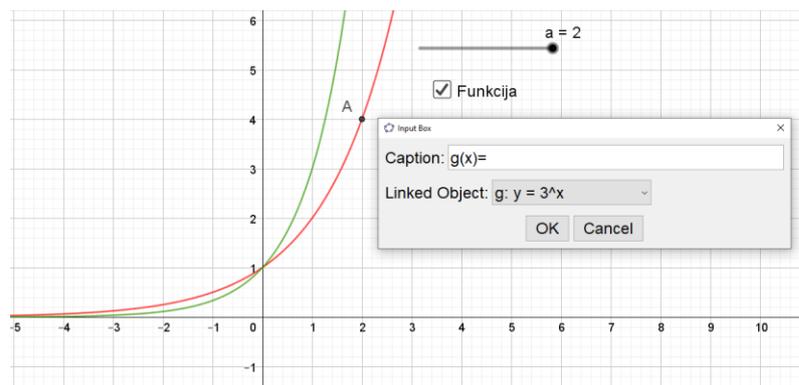
Figure 1.7: Option "Input Box".

After that, this text field also appears in Figure 1.8. By changing the value in that field, the value of the function g will also change. Generally, this option allows the user to change the values of already defined objects by changing values of these fields.
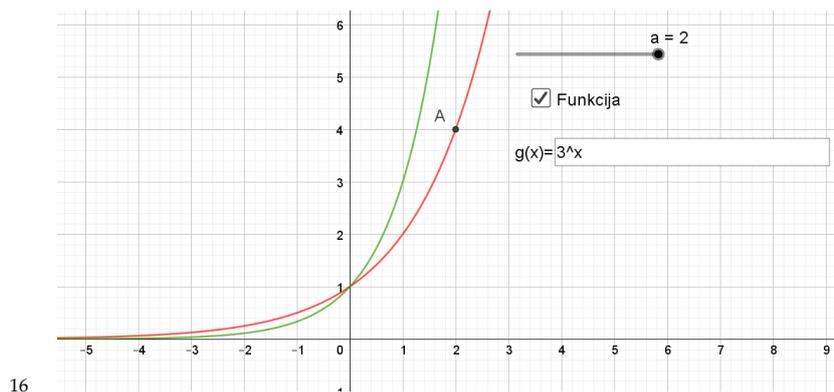
Figure 1.8: View of field "Input Box".

Wolfram Mathematica

Wolfram Mathematica (skraćeno Mathematica) jeste matematički softver (programski jezik i paket) za sim-boličko i numeričko rešavanje problema iz svih naučnih disciplina, u kojima je matematika neophodna za modeliranje i rešavanje. Tvorac ovog softvera je Stephen Wolfram, a dalje ga razvija kompanija Wolfram Research. Za razliku od GeoGebre, Mathematica nije besplatna, pa samim tim i nije lako i široko dostupna. Sa druge strane, Mathematica pruža mnogo više mogućnosti od GeoGebre za različite veoma složene postupke, a može se iskoristiti i za vizeulizaciju mnogih problema, kako bi studenti mogli lakše da ih razumeju. Ovde ćemo posebnu pažnju obratiti na dve ugrađene funckije koje nam omogućavaju dinamički prikaz problema:

– Manipulate;

– Animate.

The function "Manipulate" allows to set slider. Let's show it on the same example as with GeoGebra. "Manipulate" requires two arguments `Manipulate[expressions,{k,a,b,step}]`. The first is a set of expressions, and the second represents the value of k which will be displayed on the slider and which will take values from a to b with a step "step".

We will now use this option to draw points from the graphs of the function $y = 2^x$. The following functions will be used:

– `Table` which creates a list - in our case a set of points $(a, 2^a)$.

– `ListPlot` which draws a given set of points. Here we will use two additional options for drawing graphics. Option `PlotStyle->PointSize[ ]` gives us the ability to specify the size of the points we draw (we enlarge them if we want to emphasize them further). Second option we will use is `PlotRange`, by which we will define the interval on which we draw the graph (the first pair represents values on the x-axis, while the other gives the interval on the y-axis to be displayed).

On Figure 2.1 we can see that by using the mentioned functions we get a slider n which represents the number of points which we draw. By moving this counter to the right we get an increasing number of points from the graphics of the function $y = 2^x$ and we can understand better the definition of the graphics. In the given example, increment for the number n is 0.1.

In[1]:= `Manipulate[ListPlot[Table[{a, 2^a}, {a, -5, n, 0.1}], PlotStyle → PointSize[0.01],`
`PlotRange → {{-5, 2}, {0, 4}}], {n, -5, 2, 0.1}]`

Out[1]=
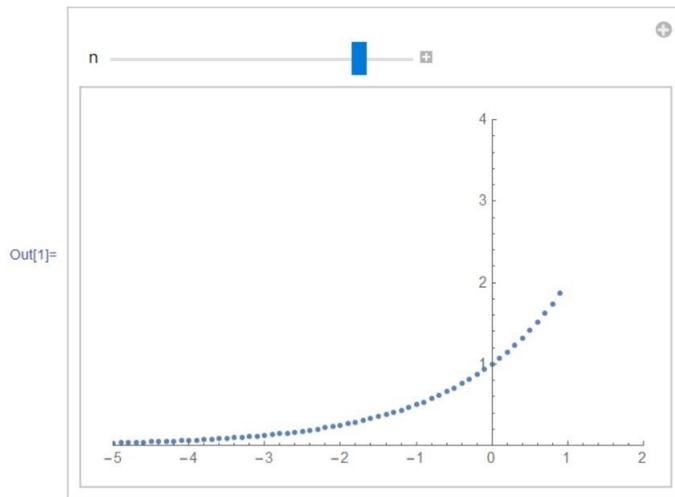
17

18

Figure 2.1: Manipulate.

The function "Animate" has the same call, i.e. the same arguments. The only difference is that by executing these commands the corresponding animation has been immediately starting, which we can change with the buttons shown in Figure 2.2. We can stop, slow down, or speed up the animation. We can also change the direction of the slider (descending or ascending).

In[1]:= `Animate[ListPlot[Table[{a, 2^a}, {a, -5, n, 0.1}], PlotStyle → PointSize[0.01],`
`PlotRange → {{-5, 2}, {0, 4}}], {n, -5, 2, 0.1}]`

Out[1]=
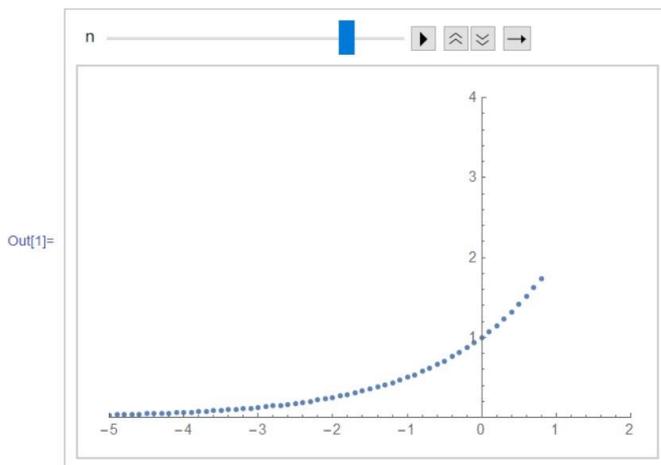
19

20   Figure 2.2: Animate.

Application of GeoGebra and Mathematica in visualization of some complex problems

In this chapter, through three more complex examples, we will show how a problem can be better visual- ized and therefore it is easier to understand and solve. Lecturers will then be able to create similar animations for many other mathematical problems.

Example 3.1 One of the biggest problems for first-year university students is understanding the definition of a convergent sequence

$$\lim_{n\to\infty} a_n \Leftrightarrow (\forall \varepsilon > 0)(\exists n_0 \in N)(n \geq n_0 \Rightarrow |a_n - a| < \varepsilon).$$

We will try, through animations in GeoGebra and Mathematics, to make it easier to understand this definition.

For a start, we consider $a_n = \frac{1}{n}$. It is clear that for all ε, we easily get that $n_0 = \lfloor 1/\varepsilon \rfloor + 1$. We will set the slider for ε, a crtaćemo niz pomoću naredbe "Niz". and we will draw a string using the command "Sequence". This command allows us to create a sequence of the points $\{(k, a_k)|k = 1,2,...,m\}$. We choose the value m so that it is slightly larger than n_0 to see not only first point enters the ε- neighbourhood, but also the next few that are in the same neighbourhood.. In our applet we took $m = \lfloor 1/\varepsilon \rfloor + 5$, Figure 3.1. By moving the slider, by changing the value ε the ε-neighbourhood of point 0 also changes. Students can see that this sequence, regardless of value ε,, starting with n_0, will be in that neighbourhood.
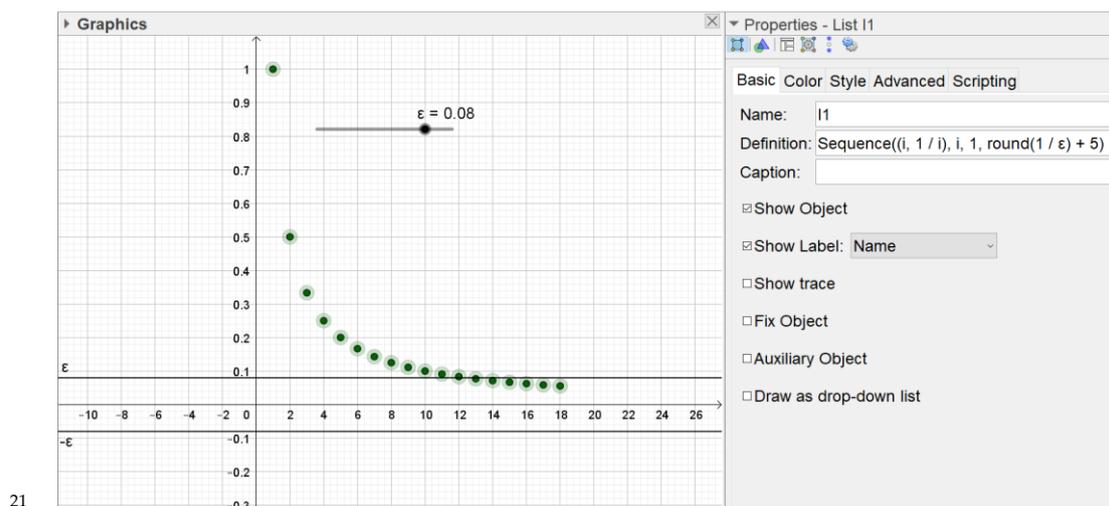
Figure 3.1: Sequence $a_n = \frac{1}{n}$.

Let us also consider a nonmonotonic sequence $a_n = \frac{(-1)^n}{n}$. To show the dependence betweenε i n_0, we will construct two sliders for these values, Figure 3.2. In this way we can change both the ε-neighbourhood of the point 0 and the number points of the sequence that we want to show to the students. Of course, it is very easy to create a similar applet for any other sequence and another ε-neighbourhood of some point.

22
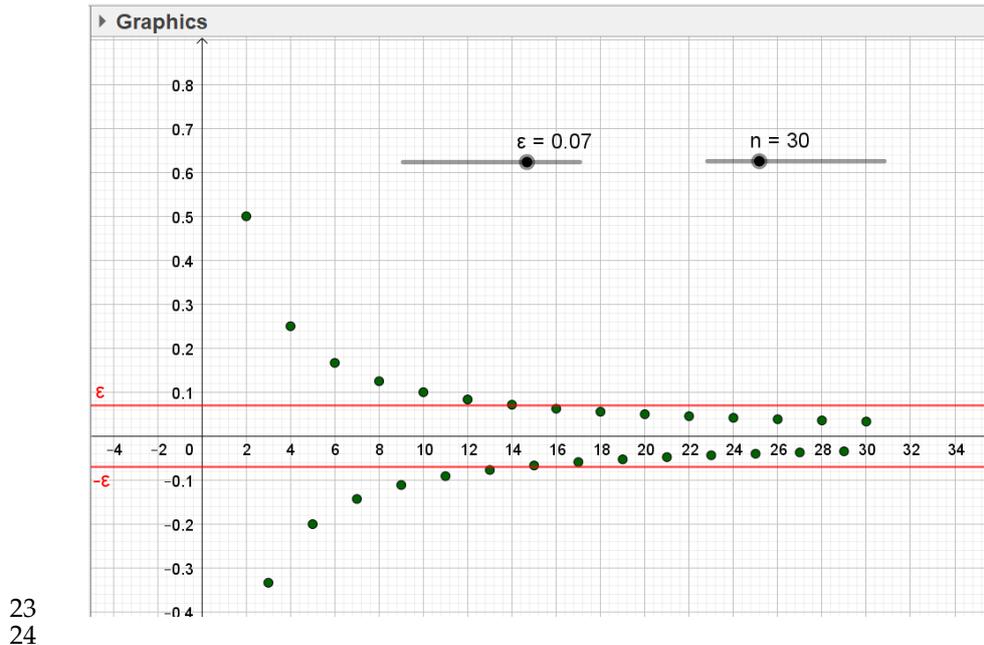
Figure 3.2: Sequence $a_n = \dfrac{(-1)^n}{n}$

Finally, we will create a similar construction in Mathematica. The command "Show" will allow us to show the ε-neighbourhood (using functions $y = \varepsilon$ and $y = -\varepsilon$) and set of the points that represent the elements of the sequence $a_n = \dfrac{(-1)^n}{n}$. For each slider, we can display the current values and animate those – by clicking on the icon that is located at the end of the slider, Figure 3.3.

```
In[1]:= Manipulate[Show[Plot[{ε, -ε}, {x, 0, n}, PlotRange → {-0.5, 0.5}],
        ListPlot[Table[{i, (-1)^i/i}, {i, 1, n}]]], {n, 2, 200, 1}, {ε, 0.5, 0, -0.001}]
```
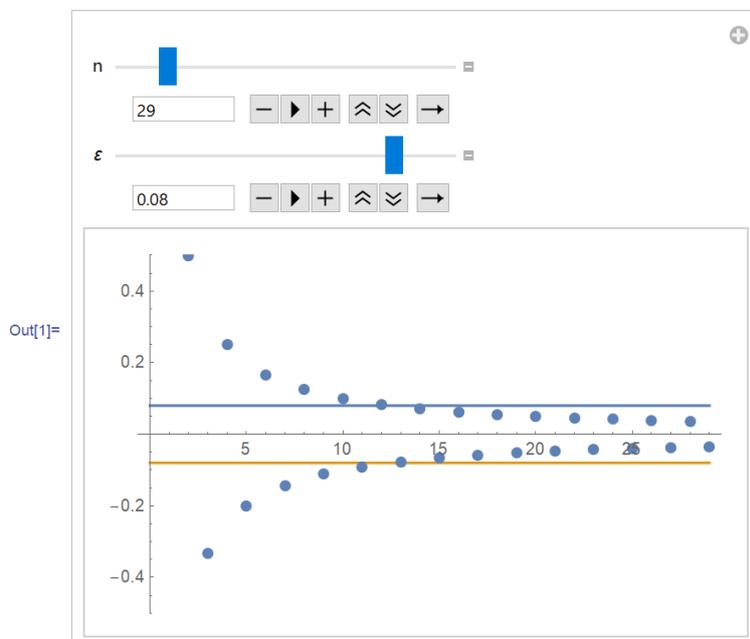

Figure 3.3: Sequence $a_n = \dfrac{(-1)^n}{n}$ in Mathematica.

Primer 3.2 When defining a definite integral, often for students, it is difficult to understand concept of Riemann sum. Let's show how in GeoGebra and Mathematica we can give a better explanation and visualization of Riemann sum.

Let's observe

$$\int_a^b f(x)dx,$$

Riemann sum for partition $\Delta\colon a = x_0 < x_1 < \cdots < x_n = b$ defined on the following way

$$26 \qquad s(f,a,b,\Delta) = \sum_{i=0}^{n-1} f(\xi_i)(x_{i+1} - x_i).$$

We will consider here only the equidistant partition

$$x_i = a + i\, h, \quad \text{gde je } h = \frac{b-a}{n}, \quad i = 0,1,\dots,n.$$

27 We will show three choices of points ξ_i:

- $\xi_i = x_i$, for each $i = 0,1,\dots,n-1$ – left rectangle;
- $\xi_i = x_{i+1}$, for each $i = 0,1,\dots,n-1$ – right rectangle;
- $\xi_i = x_i + \frac{h}{2}$, for each $i = 0,1,\dots,n-1$ – idpoint rectangle.

The command we will use is "Rectangle Sum", which we call with 5 arguments. The first is function f, and the next two are the lower and upper limits a and b for the integral. The fourth argument is the number of subintervals into which we will divide $[a,b]$. The last argument is the position of the beginning of the rectangle (for 0 – we get the left rectangles, for 1 – right, and for 1/2 – midpoint rectangles).

On example

$$\int_0^3 (3e^{-x} + x)dx.$$

we will show how the Riemann sum can be visualized. We will do this in GeoGebri. We will set up two sliders. The first is n – the number of subintervals, i.e. rectangles, and the second is $k \in [0,1]$ – position of the beginning of the rectangle. Figure 3.4 shows the left rectangles for $n = 15$ and $n = 30$. Below the graphics, the variable "rez" is shown. It represents the value of the sum of the areas of the left rectangle.

28
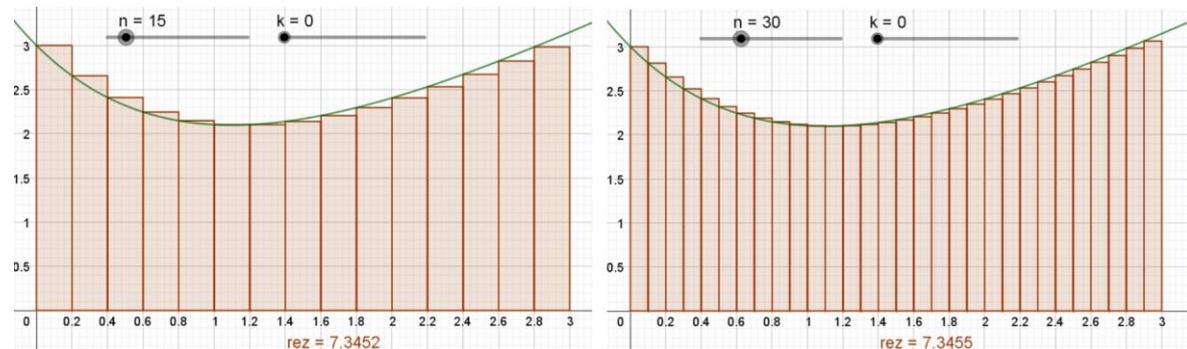
Figure 3.4: Left rectangle.

29

Similarly, we can obtain both the midpoint and right rectangles (Figure 3.5), by setting the slider k to the values $1/2$ i 1, respectively. Of course, by changing the value on the slider k we can get different rectangles. Below the graphics from Figure 3.5 we will show "rez" that represents the sum of the areas of the midpoint rectangles, or the sum of the areas of right rectangles.

30
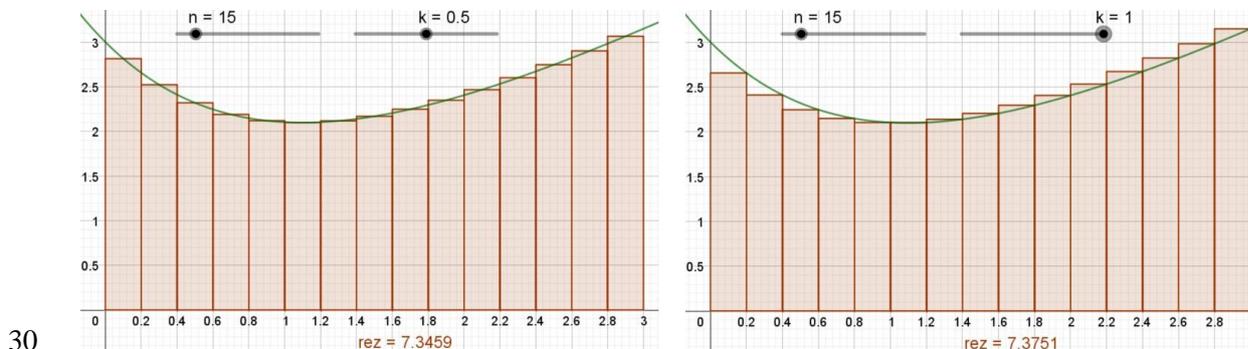
Figure 3.5: Midpoint (left) i right (right) rectangles.

In order to be able to animate the Riemann sum in Mathematica, it is necessary to program the corresponding function "rectangles", which will draw the given rectangles and calculate the integral sum. We will call this function with more arguments. The first argument is a function f, followed by an independent variable, lower and upper limit of the integral a and b, and n – the number of rectangles we draw. The penultimate argument k represents the position of the beginning of the rectangle, and the last argument the ratio of the unit lines on the x and y-axes. The whole program code is given in Figure 3.6

```
rectangles[f_, {x_, a_, b_, n_}, k_, ar_] := Module[{h, xi, xih, gr, pl, t, fp},
        b - a
    h = ───── // N;
          n
    gr = Graphics[
      Table[
        xi = a + (i - 1) h;
        xih = xi + h;
        fp = f /. x → xi + k * h;
        {
         RGBColor[1, 0.6, 0],
         Rectangle[{xi, 0}, {xih, fp}],
         RGBColor[1, 0.4, 0],
         Line[{{xi, 0}, {xih, 0}, {xih, fp}, {xi, fp}, {xi, 0}}]
         },
          {i, 1, n}] // Flatten
      ];
    Print["Result: ", {n, SetPrecision[Sum[h * f /. x → (a + i * h + k * h), {i, 0, n - 1}], 5]}];
    pl = Plot[f /. x → t, {t, a, b}];
    Show[gr, pl, Frame → True, AspectRatio → ar]
]
```

31

Figure 3.6: Program in Mathematica – Riemann sum representation (rectangles)

32

Now we can animate this sum with the function "Manipulate", and the value of the integral sum will be printed below the graphics. Figure 3.7 shows the midpoint rectangles.

```
Manipulate[rectangles[3 E^-x + x, {x, 0, 3, n}, k, 0.5], {n, 4, 50, 1}, {k, 0, 1, 0.1}]
```
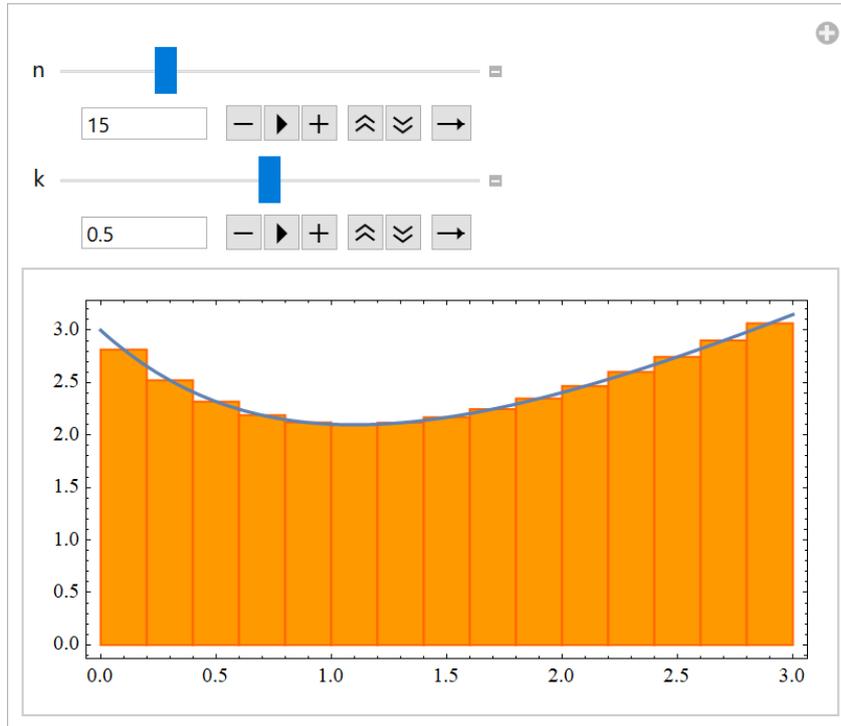


```
Result: {15, 7.3459}
```

Figure 3.7: Midpoint rectangles – Mathematica

Example 3.3 In the last example, with the help of GeoGebra and Mathematica, we will show the application of Newton's method for numerical solution of the equation $f(x) = 0$.

If x_0 is the initial iteration, then the iterations of the Newton's method are obtained as follows

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, \quad k = 0,1,2, \ldots$$

The geometric interpretation of this method is especially interesting to us and we can present it in a dynamic environment. The iteration x_{k+1} is obtained as the intersection of the tangent from the point $(x_k, f(x_k))$ to the curve $y = f(x)$ with the x-axis. Of course, it is also desirable to draw the first few iterations, which is almost impossible with the help of classical teaching aids. Therefore, we will show how this can be done, first using GeoGebra.

To get through the iterations, it is necessary to use the "IterationList" command, which has three arguments. The first argument is the function $\varphi(x)$ which creates an iterative procedure $x_{k+1} = \varphi(x_k)$, $k = 0,1, \ldots$ In our case, this function is defined by

$$\varphi(x) = x - \frac{f(x)}{f'(x)}.$$

A derivative of the function is easily obtained by using the command "Derivate". The second argument to this command is the initial iteration x_0, and the third is the number of iterations n.

We consider the equation $f(x) = 0$, for $f(x) = x^3 - 2x^2 + x + 1$. We will also use a tool "InputBox", which we will associate with this function, so that the user of this applet can change function f. We will do the same for the initial iteration x_0.

Also, we will set the slider for n and create a list of iterations, which we will display in the applet in the drop-down menu.

In Figure 3.8 we can see the first $n = 7$ iterations of Newton's method, if we open the list (picture on the left), as well as the geometric interpretation of the first iteration (picture on the right). Note that only the "First iteration" field is checked in the applet.

34
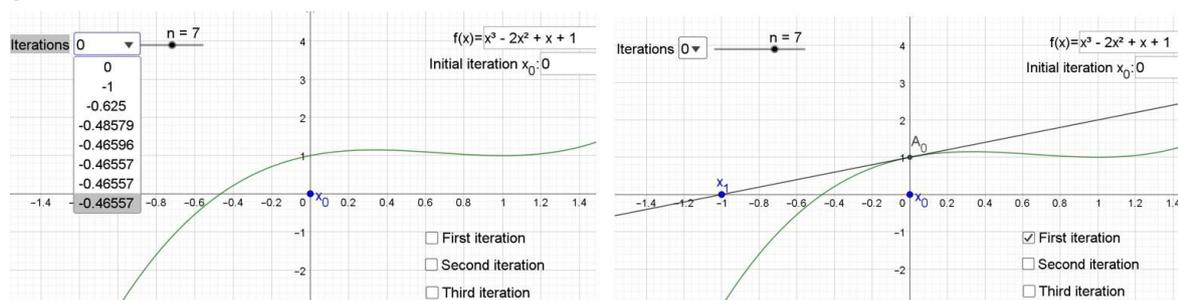
Figure 3.8: Initial i first iteration of Newton's method

35

If we check the fields "Second iteration" and "Third iteration" (Figure 3.9) we get both the second (left) and the third iteration (right), respectively.

36
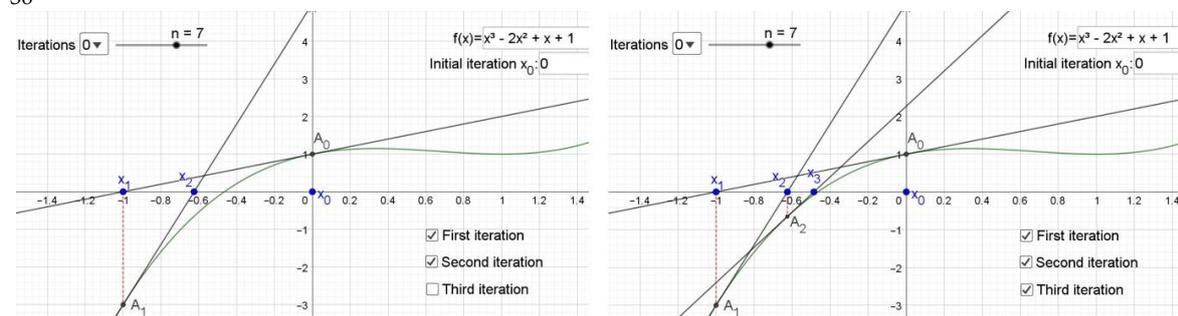
37

Figure 3.9: Second and third iteration of Newton's method

To show Newton's method in Mathematica, we write the function "newton", which will calculate iterations of this methods and show its geometric interpretation. We call this function with 5 arguments – the first is the function f, followed by the independent variable x i and the initial iteration x_0. The last two arguments represent the number of iterations n of Newton's procedure and the number of iterations $iter$ whose geometric interpretation we want to show, Figure 3.10. Also, we will use:

– built-in function for derivative – **D[·, ·]**;

– built-in function for minimum and maximum value of list – **Min[·]**, **Max[·]**;

– **Do** loop.

In addition to the above, we also used the options for color graphics functions – **RGBColor[·, ·, ·]** and line thickness – **Thickness[·].**

38

```
newton[f_, {x_, x0_}, n_, iter_] := Module[{list, points, tangents},
      it[0] = x0 // N;
      first = D[f, x];
      Do[
                          f /. x → it[k]
        it[k + 1] = it[k] - ───────────────── , {k, 0, n - 1}];
                          first /. x → it[k]
      list = Table[{k, it[k], f /. x → it[k]}, {k, 0, n}];
      points = Table[{it[k], 0}, {k, 0, iter}];
      tangents = Table[(first /. x → it[k]) * (xx - it[k]) + (f /. x → it[k]), {k, 0, iter - 1}];
      m = Min[Table[it[k], {k, 0, n}]];
      M = Max[Table[it[k], {k, 0, n}]];
      gr1 = ListPlot[points, PlotStyle → PointSize[0.025]];
      gr2 = ListPlot[Table[{it[k], f /. x → it[k]}, {k, 0, iter - 1}],
                          PlotStyle → {PointSize[0.025], RGBColor[0, 1, 0]}];
      gr3 = Plot[tangents, {xx, m, M}];
      gr4 = Plot[f, {x, m, M}, PlotStyle → {RGBColor[1, 0, 0], Thickness[0.007]}];
      Show[gr1, gr2, gr3, gr4, PlotRange → All]
      ]
```

39

Figure 3.10: Program code for Newton's method.

40

Now, we will calculate *n* iterations of this method and show the geometric interpretation for the first *iter* iterations, Figure 3.11.

41

```
Manipulate[newton[x³ - 2 x² + x + 1, {x, 0}, n, iter], {n, 1, 10, 1}, {iter, 1, 10, 1}]
```
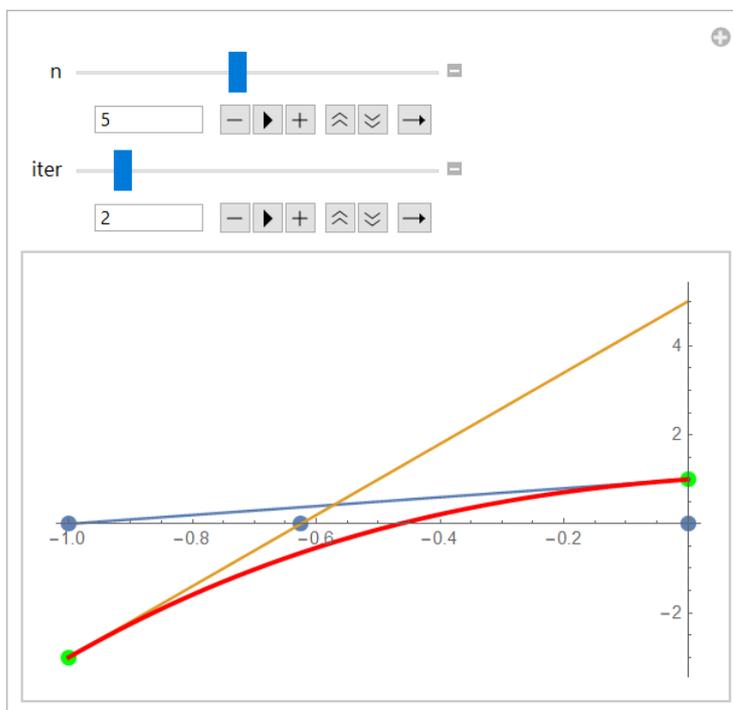

42

Figure 3.11: Newton's method in Mathematica

Finally, we will write first *n* iterations x_k, as well as the values of the function $f(x_k)$, Figure 3.12.

```
TableForm[Join[{{"k", "x[k]", "f[x[k]]"}}, lista]]
```

k	x[k]	f[x[k]]
0	0.	1.
1	-1.	-3.
2	-0.625	-0.650391
3	-0.485786	-0.0724016
4	-0.465956	-0.00135174
5	-0.465571	-5.02402×10^{-7}

Figure 3.12: First *n* iterations of Newton's method

References

[1] Krejić, N., Herceg, Đ . "Matematika i mathematica", Departman za matematiku i informatiku, PMF, Novi Sad, 2004.

[2] Ljajko, E. "Uticaj GeoGebra-e na predavanje i učenje analitičke geometrije u srednjoj školi", doktorska disertacija, Univerzitet u Novom Sadu, Prirodno-matematički fakultet, Novi Sad, 2014.

[3] Prentović, B. "Računar u nastavi analitičke geometrije u gimnaziji", doktorska disertacija, Univerzitet u Novom Sadu, Prirodno-matematički fakultet, Novi Sad, 2015.

[4] https://wiki.geogebra.org/en/Tutorials

Author:
Đurđica Takači

created by wordificator.com

QUESTIONS:

How is blended learning created in a dynamic learning environment?

How can the surface area and the body volume in a vector dynamic learning environment be determined?

How can the lateral area and the volume of solid of revolution be determined by using a definite integral in a dynamic learning environment?

Mathematics contents in dynamic geometry environment

Contents

Introduction

Blended learning is created by the integration of new information and communication technologies, ICT, into the teaching process ([1], [2]). In the late 1990s and early 21st centuries, blended learning developed intensively with the advent of the Internet and the World Wide Web ([8], [9]). But during the Covid 9 crisis it became the only learning method in the world.

There are different definitions of blended learning in the literature (for example [1], [2]). By the term blended learning we will understand: distance learning, E-learning, as well as face-to-face learning. The basic characteristics of distance learning are the physical distance between teacher and student ([6]). Qualitative distance learning supposes a developed ICT environment and practically cannot be applied without it. Distance learning also means e-learning or "online" learning, because it is associated with the use of different media in the teaching process. On the other hand, learning in the classroom (face to face) today is increasingly taking place in a modern ICT environment.

In this paper, the teaching materials for blended learning of the mathematics course at almost all faculties and universities around the world have been prepared and presented. They can also be used for distance learning, as well as in the classroom, but in both cases good internet is needed. This means that the teaching materials are prepared for "online" learning.

The author has taught at various mathematics courses (at various universities in Serbia and abroad) and gained a lot of experience about the cognitive conflicts that occur in learning these courses.

All teaching materials have been prepared methodically, with the aim that students better understand the teaching contents and eliminate cognitive conflicts that arise during students' learning. All processed mathematical contents are visually representative, i.e. their graphical representations are presented in addition to the corresponding algebraic and verbal ones. The author has been using the *GeoGebra* software package for teaching mentioned contents until the beginning of 2020. Since then, the author has exclusively used *GeoGebra* for blended learning, preparing different applets for each lesson. This paper presents the applets that (according to the author) contribute the most to the elimination of cognitive difficulties in the processing of teaching mathematics contents. The applets are adapted for both teachers and students and the prepared materials can be presented directly to students without prior corrections, but they can also be corrected and adapted to the needs of users.

About 40 dynamical applets are created and export as Web page to "*GeoGebra Material*", to the folder TECOMP-aplets, where they can be opened and their dynamic properties can be used for teaching and learning. *GeGebr*a applet can be opened with click on figure or corresponding text. Within an open applet, sliders can move, which simultaneously causes changes in the corresponding graphical, verbal, and algebraic representations of mathematical objects. The applet can also be opened in *GeoGebra*, so that all considered objects can be changed, such as vectors, functions, operations, etc., and thus adapted to the immediate needs of the user.

The applets contain two graphics views. One of them is used as a whiteboard where verbal and algebraic representations of mathematical objects were written, based on data obtained using *GeoGebra*. The other graphics view is used for graphics representations of the corresponding objects were shown. The text in *GeoGebra* is written in TEX, (only a few simple commands are used, and there are a lot of commands given in "Symbols".)

In this paper the selected mathematical contents and their geometric interpretations are presented in which visualization and dynamic properties enable their better understanding. The first part deals with vectors and their application to geometric problems. The second part deals with the definition of a definite integral and its application to the calculation of the area of plane curves. The plain curves appearing in the integrals are given in algebraic and polar coordinates, as well as by parameters. Finally, the calculation of the lateral area and volume of the solids of revolution, arising by rotations of curves around x- axes (or y- axes), which are shown in the 3D graphic view, is processed, and by activating the applet, it is possible to monitor their changed and transformation.

Vectors in dynamic environment

–Vectors in Cartesian coordinate system

A Cartesian coordinate system is given with the axes $x, y,\ z$, and the origin $O(0,0)$. The points $A(1,0,0)$, $B(0,1,0)$ and $C(0,0,1)$ are given and the unit vectors $\vec{\imath},\ \vec{\jmath}, \vec{k}$ are defined as

$$\vec{\imath} = \overrightarrow{OA},\ \ \vec{\jmath} = \overrightarrow{OB},\ \ \ \vec{k} = \overrightarrow{OC}.$$

In Figure 1., right graphic view is used for 3D graphics. The points A, B, C and unit vectors $\vec{\imath},\ \vec{\jmath}, \vec{k}$, are presented in violet color.

For each vector $\vec{a}$, there exists unique point $M(a_x, a_y, a_z)$ such that

$$\overrightarrow{OM} = \vec{a} = a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k}.$$

Left graphic (Figure 1.) view is used for algebraic and verbal representations. The sliders a_x, a_y, a_z with values $3, 4, 5$, corresponds to the coordinates of vector $\overrightarrow{OM}$. The point $M(a_x, a_y, a_z)$ is showen on the graphs (graphic view 3D) (Figure 1.), together with the vector $\overrightarrow{OM}$, and its coordinates $\overrightarrow{OM_x} = a_x\vec{\imath}$, $\overrightarrow{OM_y} = a_y\vec{\jmath}$, $\overrightarrow{OM_z} = a_z\vec{k}$, in blue color.

The magnitude of vector $\overrightarrow{OM}$ denoted as $|\overrightarrow{OM}| = |\vec{a}|$ is given by

$$|\overrightarrow{OM}| = |\vec{a}| = |a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k}| = \sqrt{(a_x)^2 + (a_y)^2 + (a_z)^2}.$$

Graphic view 3D in *Geogebra* is used for presented vectors. The main *Geogebra* commands in this view are almost the same as the commands for graphic view 2D. For example, the vector will be drawn when after command "vector" we one connect two points.

In Figure 1., *Geogebra* applet, TECOMP- -INTR- vectors which can be obtain by the click on the picture or its label is shown. The displacement of a vector in 3D space can be dynamically monitored tracking off the changes of its components given by the corresponding sliders.
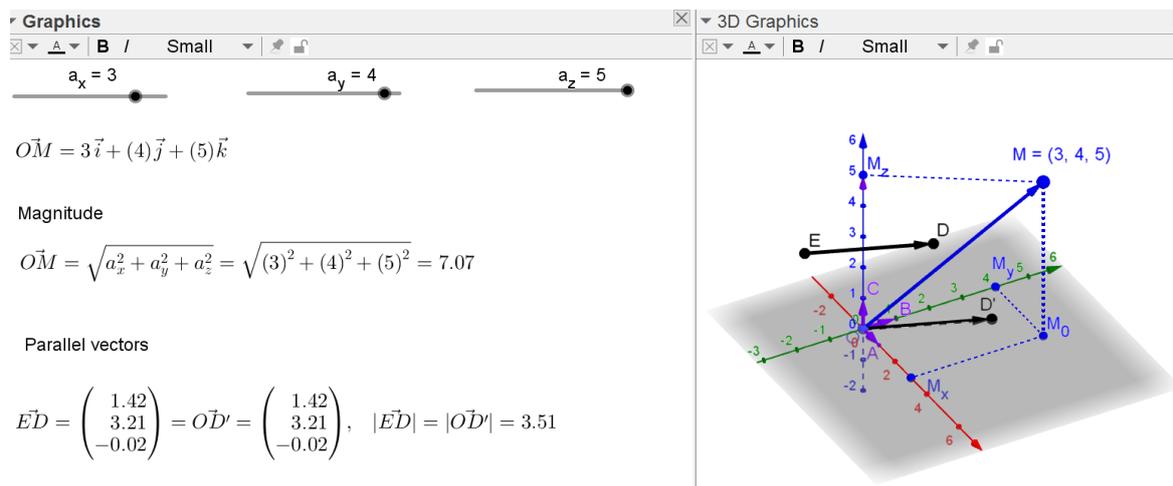
Figure 1.

The changes of vector magnitude $|\overrightarrow{OM}|$, caused with change of its components can be monitored. The applet can also be used in order to derive the vector intensity formula.

It can be visually demonstrated that parallel vectors of the same intensities and the same directions have the same coordinates, no matter where they are located in space. In particular, if the vector $\overrightarrow{OD'}$ is given, and the vector $\overrightarrow{ED}$ is obtained by its parallel dispayment, then the vectors $\overrightarrow{OD'}$ and $\overrightarrow{ED}$ have the same coordinates.

−Addition of vectors and multiplication of vectors by scalar

The sum of two vectors $\overrightarrow{OM} = a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k}$ and $\overrightarrow{ON} = b_x\vec{\imath} + b_y\vec{\jmath} + b_z\vec{k}$, is the vector

$$\overrightarrow{OD} = \overrightarrow{OM} + \overrightarrow{ON} = (a_x + b_x)\vec{\imath} + (a_y + b_y)\vec{\jmath} + (a_z + b_z)\vec{k}.$$

Geometrically, the addition of two vectors is interpreted such that the end point of the first vector is the starting point for the second vector. A vector whose beginning is the starting point of the first vector, and the end of the end point of the second vector is their sum.

In Figure 2., the applet TECOMP-sum-vectors is shown. On the left graphic view the sliders a_x, a_y, a_z, and b_x, b_y, b_z, with corresponding values $3, 4, 4,$ i $4, 1, 1,$ i.e., the components of vectors $\overrightarrow{OM}$ i $\overrightarrow{ON}$, respectively, as well as the vectors $\overrightarrow{OM}, \overrightarrow{ON}$ and vector $\overrightarrow{OD} = \overrightarrow{OM} + \overrightarrow{ON}$, algebraically are representing.

On the right graphic view 3D, (Figure 2.) the graphics of vectors $\overrightarrow{OM}, \overrightarrow{ON}$ and their sum, vector $\overrightarrow{OD}$ are presented, as well as the geometric addition of the vectors $\overrightarrow{OM}$ and $\overrightarrow{ON}$. The addition and subtraction of the vector by using parallelogram can be shown by activating the "parallelogram" button. The visualization of algebraic and geometry representations of multiplications of vector $\overrightarrow{OM}$ with the scalar p can be monitored.
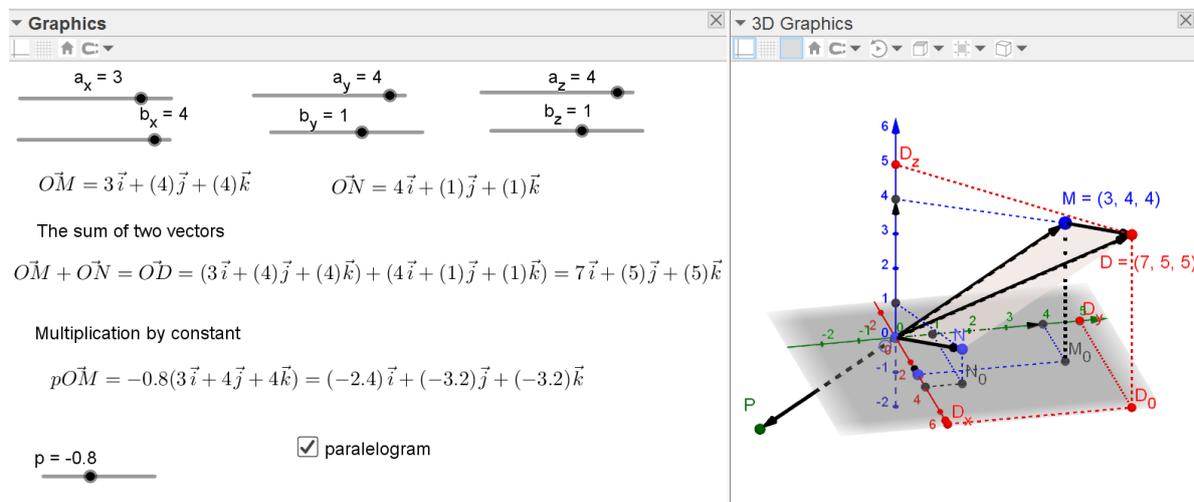
Figure 2.

By using the same applet (Figure 2.) the properties of the

– vector addition (set of vectors with addition forms a commutative group);

– multiplying a vector by a real number

visually can be shown.

– Scalar (dot) product of vectors

Scalar or dot product of two vectors $\overrightarrow{OM} = a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k}$ and $\overrightarrow{ON} = b_x\vec{\imath} + b_y\vec{\jmath} + b_z\vec{k}$ is defined as:

$$\overrightarrow{OM} \cdot \overrightarrow{ON} = |\overrightarrow{OM}| \cdot |\overrightarrow{ON}| \cos \alpha,$$

where α is the angle between the vectors $\overrightarrow{OM}$ and $\overrightarrow{ON}$.

Figure 3., shows the image of the *Geogebra* applet, TECOMP-scalar-product, which is obtained by clicking on the image or image tag. The changes of the scalar product of vectors $\overrightarrow{OM}$ and

$\overrightarrow{ON}$, caused by changes in their coordinates, given by the sliders, can be monitored dynamically. The scalar product can be also calculated as:

$$\overrightarrow{OM} \cdot \overrightarrow{ON} = a_x b_x + a_y b_y + a_z b_z.$$

The dependence of the scalar product on the angle between the vectors $\overrightarrow{OM}$ i $\overrightarrow{ON}$ can be monitored, also. For example, the angle between the vectors $\overrightarrow{OM} = 3\vec{\imath} - \vec{\jmath} + 2\vec{k}$ and $\overrightarrow{ON} = 2\vec{\imath} + 2\vec{\jmath} - 2\vec{k}$ is 90°, and their scalar product is equl to zero. It can be noted that the sign of a scalar product depends on whether the angle is acute or obtuse, because the cosines of such angles have different signs.

By using the same applet scalar product properties can be visually shown:

- $\overrightarrow{OM} \cdot \overrightarrow{ON} = |\overrightarrow{OM}|^2$;

- commutativity.

Also, the third vector $\overrightarrow{OP}$ can be introduced and it can be visually shown:

- $\overrightarrow{OM} \cdot (\overrightarrow{ON} + \overrightarrow{OP}) = \overrightarrow{OM} \cdot \overrightarrow{ON} + \overrightarrow{OM} \cdot \overrightarrow{OP}.$
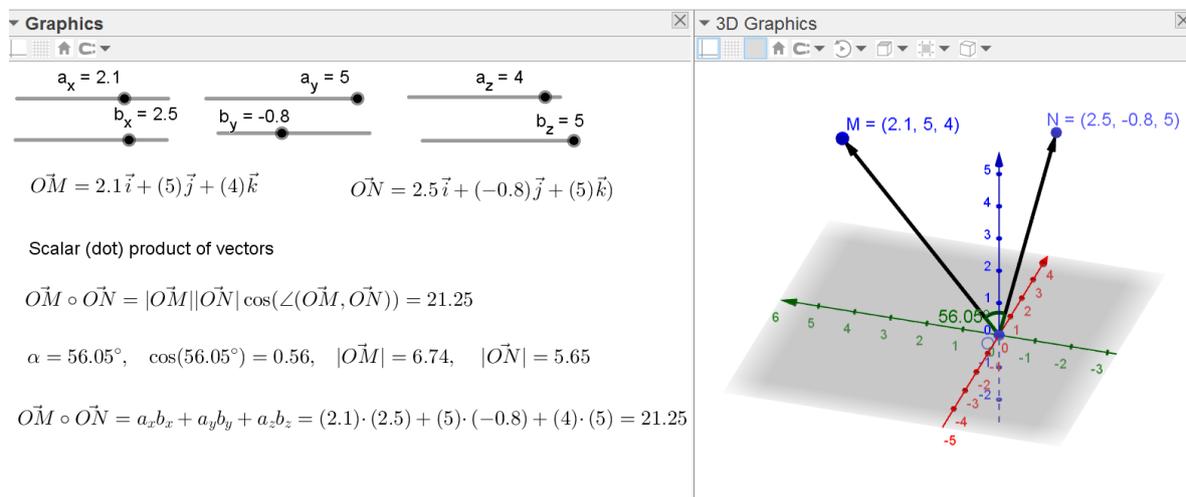
Figure 3.

−Vector (cross) product of two vectors

Vector or cross product of two vectors $\overrightarrow{OM}$ i $\overrightarrow{ON}$, which are different from zero vector is the vector $\overrightarrow{OD} = \vec{d}$, dentoted as $\vec{d} = \overrightarrow{OM} \times \overrightarrow{ON}$, which:

- is perpendicular to both vectors $\overrightarrow{OM}$ and $\overrightarrow{ON}$, and thus normal to the plane containing these two vectors;

- direction is determined by the right hand rule;

- magnitude is determined as $|\vec{d}| = |\overrightarrow{OM}| \cdot |\overrightarrow{ON}| \sin \alpha$, where α is the angle between the vectors $\overrightarrow{OM}$ i $\overrightarrow{ON}$ and it is equal to the surface area of parallelogram determined by the vectors $\overrightarrow{OM}$ i $\overrightarrow{ON}$.

Vector product of two vectors $\overrightarrow{OM} = a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k}$ and $\overrightarrow{ON} = b_x\vec{\imath} + b_y\vec{\jmath} + b_z\vec{k}$ can be determined by the relation:

$$\vec{d} = \overrightarrow{OM} \times \overrightarrow{ON} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_z & b_z \end{vmatrix}$$

Figure 4. shows the image of the *Geogebra* applet, TECOMP-vektorski_proizvod , which is obtained by clicking on the image or image tag. The vector $\vec{d}$, the vector product of the vectors $\overrightarrow{OM}$ and $\overrightarrow{ON}$, is perpendicular to the plane formed by the same vectors with the coordinates given by the sliders. By changing the coordinates of the vectors, the vector product and the area of the parallelogram that form these two vectors is changed. The conditions for the collinearity of these two vectors can also be examined by using the same applet.
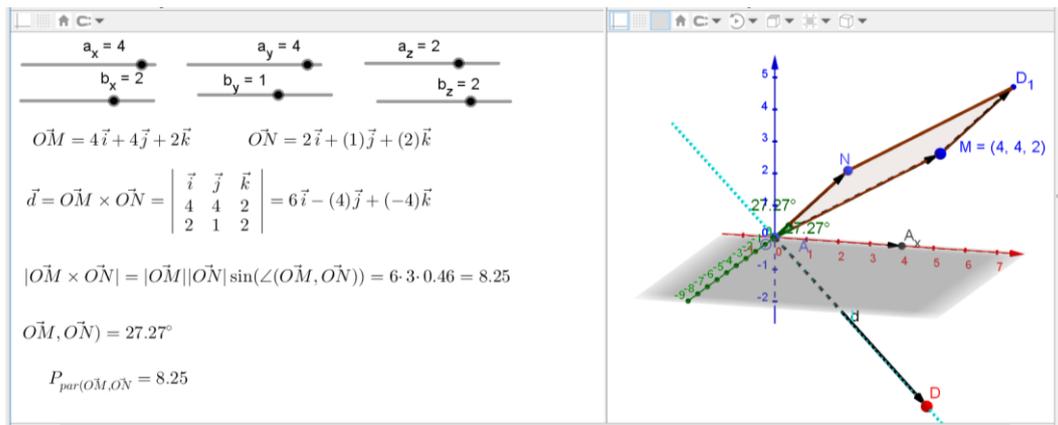
Figure 4.

By introducing a vector $\overrightarrow{OP}$ and scalar k (can be given by slider) the properties of a vector product can be visually demonstrated:

- $\overrightarrow{OM} \times \overrightarrow{ON} = -(\overrightarrow{ON} \times \overrightarrow{OM})$;
- $(k\overrightarrow{OM}) \times \overrightarrow{ON} = k(\overrightarrow{OM} \times \overrightarrow{ON})$;
- $\overrightarrow{OM} \times (\overrightarrow{ON} + \overrightarrow{OP}) = \overrightarrow{OM} \times \overrightarrow{ON} + \overrightarrow{OM} \times \overrightarrow{OP}$;
- $\overrightarrow{OM} \times (k\overrightarrow{OM}) = 0$.

−Mix product (scalar triple product)

Mixed product or scalar triple product of three vectors $\overrightarrow{OM}$, $\overrightarrow{ON}$ i $\overrightarrow{OP}$, i.e., $(\overrightarrow{OM} \times \overrightarrow{ON}) \cdot \overrightarrow{OP}$ is determined as:

$$(\overrightarrow{OM} \times \overrightarrow{ON}) \cdot \overrightarrow{OP} = |\vec{d}| \cdot |\overrightarrow{OP}| \cos \alpha,$$

where $\vec{d}$ is the vectors product $\overrightarrow{OM} \times \overrightarrow{ON}$, and α is the angle between the vector $\vec{d}$ and the vector $\overrightarrow{OP}$.

Figure 5. shows the image of the *Geogebra* applet, TECOMP-mesoviti_proizvod , which is obtained by clicking on the image or image tag. On graphic view 3D the vectors $\overrightarrow{OM} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$, $\overrightarrow{ON} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$ and $\overrightarrow{OP} = c_x\vec{i} + c_y\vec{j} + c_z\vec{k}$, are shown together with parallelepiped determined by three vectors $\overrightarrow{OM}$, $\overrightarrow{ON}$ i $\overrightarrow{OP}$. The body volume of parallelepiped

determined by *GeoGebre* is given in the left graphic view. The mixed product was also calculated directly by using the relation:

$$(\overrightarrow{OM} \times \overrightarrow{ON}) \cdot \overrightarrow{OP} = \begin{vmatrix} a_x & a_y & a_y \\ b_x & b_y & b_z \\ c_x & c_z & c_z \end{vmatrix}.$$

Vector $\vec{d}$, vector product of vectors $\overrightarrow{OM}$ i $\overrightarrow{ON}$, as well as the andle α between the vectors $\vec{d}$ and $\overrightarrow{OP}$ are algebraically and graphically shown. By changing the coordinates of these three given vectors, which are given with sliders, the parallelepiped which is formed by these three vectors, its volume and their mixed product change. The change of angle α can be used to discuss the sign of the mixed product. The conditions of complanarity of these three vectors can also be analyzed.
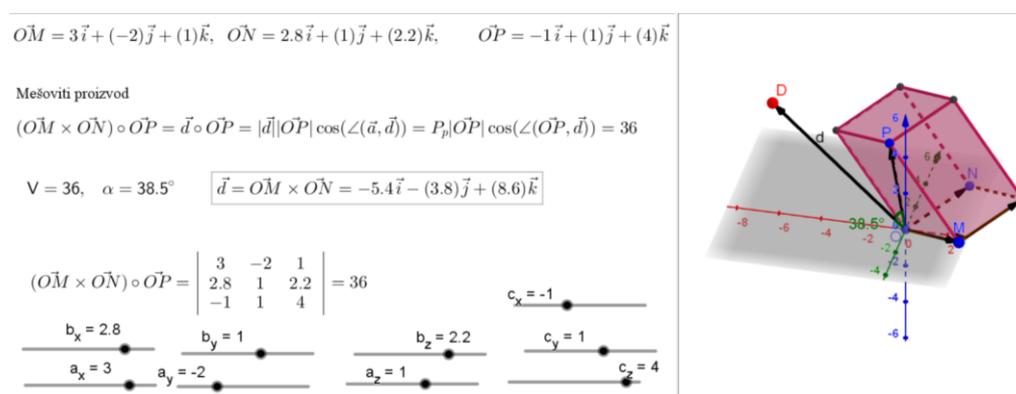
Figure 5.

[Figure 6](#). shows the image of the *Geogebra* applet which can be activated by a click, where the coordinates of the vectors depend on only one parameter and it is much easier for composing tasks that are often given in written exams. A detailed view of this applet is given via [videoklip](#).
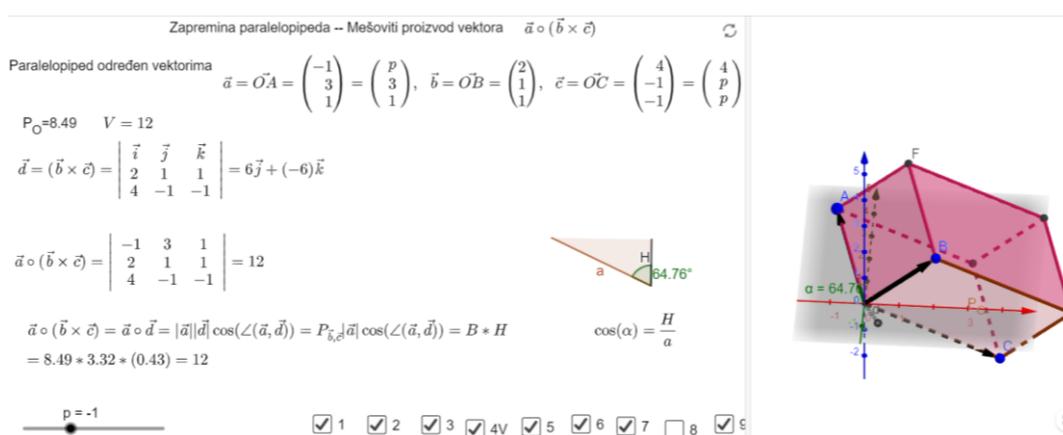
Figure 6.

Task: The vectors $\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$, $\vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}$ and $\vec{c} = c_x \vec{i} + c_y \vec{j} + c_z \vec{k}$ are given.

- Determine the vector $\vec{a} = \frac{\vec{a}}{2} + 2\vec{b} - \vec{c}$;

- Determine the surface area of the triangle forming the vectors $\vec{a}$ and $\vec{c}$;

- Determine the height corresponding to the side $\vec{a}$ of the parallelogram formed by the vectors $\vec{a}$ and $\vec{b}$;

115

- Determine the volume of a parallelepiped formed by vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$;
- Determine the equation of a plane that is parallel to the plane formed by the vectors $\vec{a}$ and $\vec{b}$, and which contains a point $A(x_0, y_0, z_0)$.

Definite integral in dynamic environment

–The definition of definite integral

Let the function f be positiv on the interval $[a, b]$, meaning that its graph is above x −axis. Then a flat figure bounded with the interval [a, b], the ordinates of the function f at the points $x = a$, and $x = b$, and the graph of the function f over the interval [a, b], is called a curvilinear trapezoid over [a, b]. The problem of determining the area curvilinear trapezoid over [a, b], brought the definition of definite integral.

The continuous function f is given on the interval $[a, b]$. Let us divide the interval $[a, b]$ on n subintervals such that $a = x_0 < x_1 < x_2 < x_3 < \cdots < x_{n-1} < x_n = b$. Let us denote the lenth of $i-$interval with $\Delta x_i = x_i - x_{i-1}$, $i = 1, 2, \dots, n$, and $\Delta x = \max_i \Delta x_i$. Let c_i be the points from $[x_{i-1} x_i]$, $i = 1, 2, \dots, n$. If the following limit

$$\lim_{\substack{n \to \infty \\ \Delta x \to 0}} \sum_{i=1}^{n} f(c_i) \Delta x_i$$

exists for each division of interval $[a, b]$ and every choice of n points $c_i \in [x_{i-1} x_i]$, then it defines the definite or Riman integral of the function f on the interval $[a, b]$, i.e.,

$$\lim_{\substack{n \to \infty \\ \Delta x \to 0}} \sum_{i=1}^{n} f(c_i) \Delta x_i := \int_a^b f(x)dx,$$

Considered sum is called Riman sum. This sum is called upper or lower integral sum in the case when the function f has maximum or minimum at the points $c_i \in [x_{i-1} x_i]$, $i = 1, 2, \dots, n$, on corresponding subintervals, respectively.

Figure 7. shows the image of the *Geogebra* applet TECOMP-Uvod-Integral. On the left graphic view the area under the graph of function $f(x) = x^2$ over the interval $[0,1]$ is determined by definition and it is visually shown that it equal to definite inegral:

$$\int_0^1 x^2 dx = \frac{1}{3}.$$
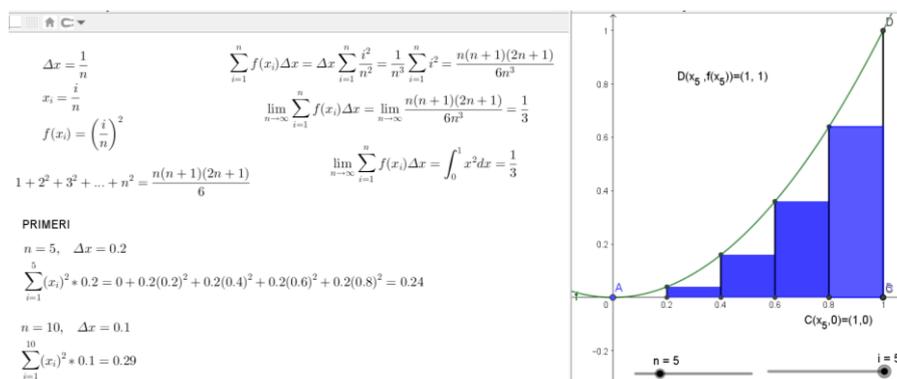
Figure 7.

On the same applet the upper and lower sums corresponding to the sums of areas of described and inscribed rectangles for given function on the given interval can be analysed. By moving the slider, increasing n one can monitor the approach of the upper and lower sums to the value $\frac{1}{3}$.

Applet TECOMP-Integral-definicija, shows the values of definite integral, upper and lower sum, obtained in *GeoGebra*, for the function $f(x) = \frac{x^p}{4}$, on the interval $[a, b]$. Activating applet one can change the values of a, b, n, as well as the value of exponent, p, of given power function. Instead of the power function, any other function can be observed.

Graphical and algebraic representations in Figure 8. are done with *GeoGebra* commands: UpperSum, LowerSum and Integral for the function $f(x) = \frac{x^3}{4}$, on the interval $[-4,0]$, for $n = 6$. The described and inscribed rectangles in corresponding curvilinear trapezoids (the areas under and above the graphs of function on the prescribed interval) are graphically shown. But their algebraic representations are not the upper and lower sums, because these sums are negative numbers. The absolute values of the upper and lower sums are equal to corresponding areas of rectangles. Compare with the example in Figure 7., where the given function is positive at the observed interval. In this case, also, n can be increasing and the behaving of the lower and upper sum of the values of the definite integral can be monitored.
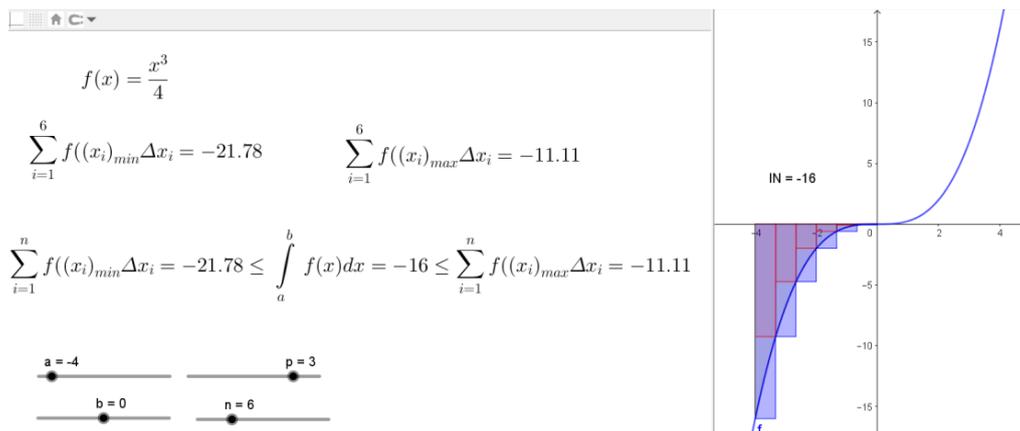
Figure 8.

Applet TECOM-Trap shows the areas of curvilinear trapezoid for the functions $f(x) = \frac{x^2}{4} + 1$ and $g(x) = x + 1$, on interval $[a, b]$ and corresponding definite integrals on the same interval. The graph of function g is line, meaning that curvilinear trapezoid is in this case is trapezoid, and derived formula is known formula for trapezoid. The area of triangle and circle can be derived by using definite integral, staring from the functions $h(x) = x$, $k(x) = \sqrt{1 - x^2}$, as well as the area of some other plane fugures.

Determining area by using definite integral is presented on videoklip.

–The properties of definite integrals

- If $a = b$, then $\int_a^a f(x)dx = 0$;

- If the function f is a constant, $f(x) = k$, $x \in [a, b]$, then $\int_a^b kdx = k(b - a)$;

- $\int_a^b f(x)dx = -\int_b^a f(x)dx;$

- If f is integrable function on the interval $[a, b]$ and $c \in [a, b]$, then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx;$$

- If f is integrable function on the interval $[a, b]$ and $f(x) \geq 0$, for all $x\ [a, b]$, then $\int_a^b f(x)dx \geq 0$, and it is equal to the area of corresponding curvilinear trapezoid;

- If f is integrable function on the interval $[a, b]$ and $f(x) \leq 0$, for all $x\ [a, b]$, then $\int_a^b f(x)dx \leq 0$, and the area of corresponding curvelinear trapezoid is equal to $|\int_a^b f(x)dx\ |$;

- If f is integrable function on the interval $[a, b]$ then $|\int_a^b f(x)dx\ | \leq \int_a^b |f(x)|dx$,

The properties of definite integrals can be visually proved by definition, i.e., by using the upper and lower sums, taking into account that the sums are not always equal to the sums of the areas of the corresponding rectangles. The applets in Figures 7. and 8. can be used for visual checking the properties of definite integral

The last property of definite integral can be visually analyzed by using the applet TECMP-Sin where definite integrals are algebraically and graphically represented

$$\int_0^{2\pi} \sin x\ dx, \int_0^{\pi} \sin x\ dx, \int_{\pi}^{2\pi} \sin x\ dx, \left|\int_{\pi}^{2\pi} \sin x\ dx\ \right|, \int_{\pi}^{2\pi} |\sin x|dx.$$

In these examples, it can be clearly seen that (in the *GeoGebra*) that the graphical representation on the graphic view 2 does not always correspond to the algebraic representation.

The following properties of definite integral:
- If the functions f and g are integrable on the interval $[a, b]$ then

$$\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx;$$

- If the functions f and g are integrable on the interval $[a, b]$ and it holds $f(x) \leq g(x),\ x \in [a, b]$, then

$$\int_a^b f(x)dx \leq \int_a^b g(x)dx,$$

can be visually monitored by using applet TECOMP-1Racun (Figure 9.). For example, the algebraic and graphical representations of definite integrals for the functions

$$f(x) = \frac{2^x}{4} + 6,\ g(x) = \frac{x(x-2)(x-4)}{4} + 2,\quad h(x) = f(x) - g(x),$$

on the intervals $[a, b]$ are calculate in *GeoGebra*. The interval can be changed by moving sliders a and b directly on applet. In algebraic view the functions f and g can be changed, too.
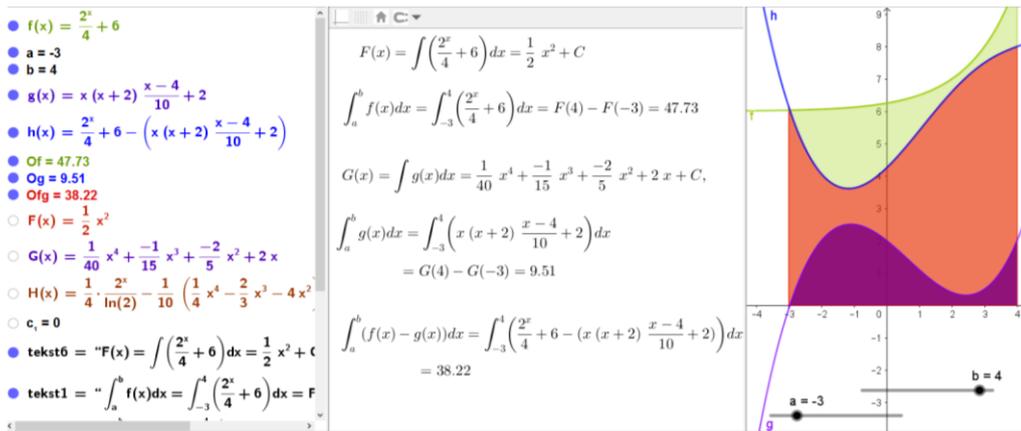
Figure 9.

Applet TECOMP-1Racun (Figure 9.) shows the algebraic view, too, besides two graphic views. The left graphic view shows verbal and algebraic representations of indefinite and definite integrals, and the right graphic view shows their graphical representation. The functions f, g and h, their indefinite integrals (functions F, G and H) and corresponding definite integrals denoted as Of, Og, Ofg, on the interval $[a, b]$ are presented. In algebraic view their algebraic representation, and in right graphic view the corresponding graphical representations can be seen. The left graphic view was used as an interactive whiteboard. For example, the notation "$F(x)$", or "$\int_a^b f(x)dx$" always remain the same, but the change of the function f, in the algebraic view, causes simultaneous change of the primitive function on the left graphic view, as well as the corresponding graphs on the right graphic view. The applet can be used for monitoring the simultaneous change of algebraic, verbal and graphic representations of corresponding objects. By our opinion the students should be shown only both graphic views, because the interval $[a, b]$ can be changed on them. The algebraic view shows where the function can be changed. Note that in the algebraic view no arbitrary constant appears in the indefinite integral, in fact $C = 0$ is taken. In algebraic view (Figure 9.), the circles next to the functions F, G and H are not colored, which means that their graphs are not shown, but they can be displayed on the applet by clicking on the circle.

–The area between the curves

If f and g are integrable functions on the interval $[a, b]$ and $f(x) \geq g(x)$, for all x $[a, b]$, then the area between the surfaces between their graphs is determined as

$$P = \int_a^b \big(g(x) - f(x)\big)dx.$$

Applet TECOMP-Površina2 (Figure 10.) shows the functions $(x) = \frac{x^2}{4} + 1$, $g(x) = x\left(\frac{4-x}{4}\right)$, $g(x) \leq f(x)$, $x \in R$, with their graphs on graphiv view 2, and the curvelinear trapezoid for each function on the interval $[a, b]$. Algebraic representations of the areas of these curvelinear trapezoids and their difference are shown on left graphic view. Interval $[a, b]$ can be changed with the sliders a and b and by changing the interval $[a, b]$ curvelinear trapezoids are changing too. The function f is positive on its domain and definite integrals for arbitrary values a and b, $a < b$ will be positive and equal to corresponding curvilinear trapezoids. Also, the case when $a > b$, can be analyzed, too.

119

$$f(x) = \frac{x^2}{4} + 1 \quad g(x) = \frac{x\,(4-x)}{4}$$

$$\int_a^b f(x)dx = \int_{-3}^{-1}\left(\frac{x^2}{4}+1\right)dx = 4.17$$

$$\int_a^b g(x)dx = \int_{-3}^{-1}\left(\frac{x\,(4-x)}{4}\right)dx = -6.17$$

$$P = \int_a^b (f(x)-g(x))dx = \int_{-3}^{-1}\left(\frac{x^2}{4}+1-\left(\frac{x\,(4-x)}{4}\right)\right)dx = 10.33$$
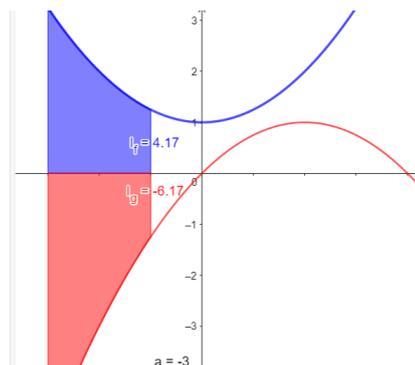
Figure 10.

The function g does not have the same sign on its domain. It is positive on the interval $[0,4]$, and negative out of that interval.

For example, on interval $[-3,-1]$ the definite integral of function f is 4.17 and it is equal to area of corresponding curvelinear trapezoid, while the definite integral of the function g is -12.33 and its absolute value is equal to area of corresponding curvilinear trapezoid. The area between mentioned graphs is 10.33 and can be obtained by using mentioned formula or calculating each area separately.

The next examples should contain functions whose differences at different intervals have different signs.

For example, on applet TECOMP-sinCos the functions $f(x) = \sin x$, and $g(x) = \cos x$ are condered. On the intervals $[0, \pi/4]$ i $[5\pi/4, 2\pi]$ it holds $f(x) < g(x)$, while on intervals $[\pi/4, 5\pi/4]$ $f(x) > g(x)$. Therefore

$$\int_0^{2\pi}\big(f(x)-g(x)\big)dx = 0, \quad \int_0^{\pi/4}\big(f(x)-g(x)\big)dx = \sqrt{2}-1,$$

and so on. The intervals and functions can be changed on the applet, but the corresponding algebraic representations on the left graphic view must also be changed.

−Areas in parametric form

The function $y = f(x)$, $x \in [a,b]$, can be given in parametric form:

$$x = g(t), \; y = h(t), \; t \in [\alpha, \beta], \; g, h: [\alpha, \beta] \to R,$$

where g is strictly monotone function and it holds

$$a = g(\alpha), \; b = h(\beta), \; y = h\big(g^{-1}(x)\big) = f(x),$$

where g^{-1} is inverze function for the g.

If the function $y = f(x)$ is given on the interval $[a,b]$ in parametric form, so that the function g is differentiable, then the area of the curvilinear trapezoid determined by the graph of the function f is determined as

$$P = \int_a^b y\,dx = \int_\alpha^\beta h(t)g'(t)dt.$$

Parametric equation of cycloid is given as

$$x = a(t - sin(t)), \qquad y = a(1 - cos(t)), \qquad a > 0, \qquad t \in R.$$

Parametric equation of astroid is given as

$$x = a\, sin^3(t), \qquad y = a\, cos^3(t), \qquad a > 0, \qquad t \in R.$$

Let us remark that both functions $x(t)$ i $y(t)$ defining astroid are periodic with the periodom 2π and therefore it is enough to consider interval $[0, 2\pi]$. The function $x(t)$ defining cycloid is not periodic but $y(t)$ is periodic one.

In package *GeoGebra* the graph of function, given in parametric form, can be obtained by introducing parameter a and by using the command:

„ Curve(<Expression>, <Expression>, <Parameter Variable>, <Start Value>, <End Value>)“,

where expressions are functions x i y.

In particular, cycloid can be inputted as „Curve($a(t - sin(t)), a(1 - cos(t)), t, 0, 2\pi$)“.

The TECOMP-Cycloid-Astroid (Figure 11.) applet shows graphs of the cycloid on the interval $[0, 2\pi]$ and the astroid, the area between the cycloid and x −axis, as well as the area of the figure formed by the astroid. Therefore the function F depends on x, not on t, which is shown in the lower part of the graphic view, because *GeoGebra* only recognizes a function with variable x.
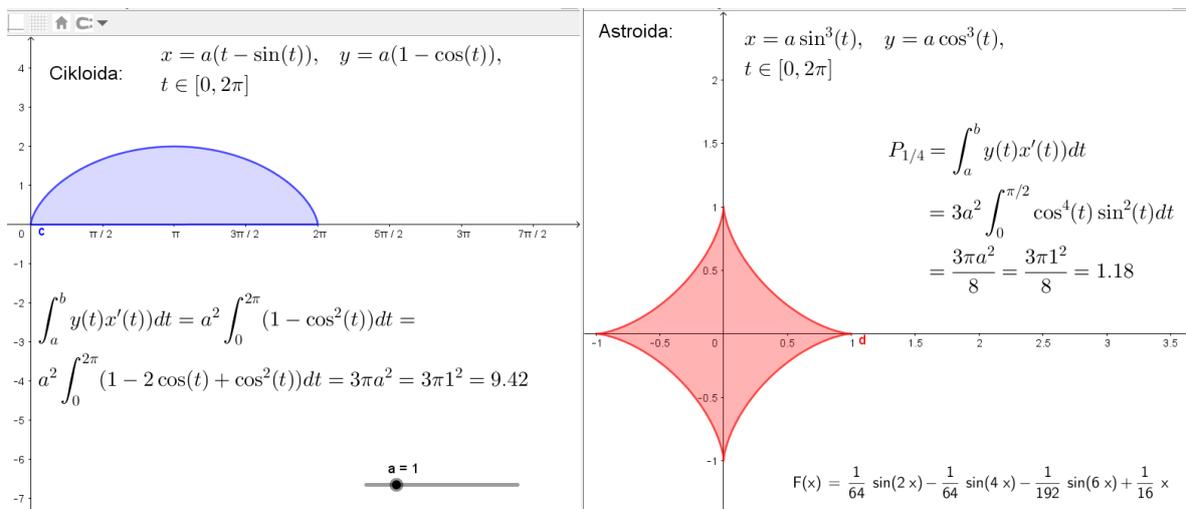
Figure 11.

−Areas in polar coordinates

Let the coordinate origin be marked with O in Cartesian rectangular coordinate system and let $A\,(x, y) \neq O\,(0,0)$ be a point in the xy −plane. Denote by r the distance of point A from the origin, and α the angle between OA and the positive direction of the x −axis. The numbers α and r, thus determined, are called the polar coordinates of the point A. The relationship between the polar and Cartesian coordinates is given by the relations

$$x = r\, cos\,\alpha, \quad y = r\, sin\,\alpha.$$

Thus, point A in the Cartesian coordinate system is determined by the rectangular coordinates x and y and the polar coordinates r and α. Polar coordinates can be used to define functions by providing a link between r and α, $r = f(\alpha)$, where $f: (\alpha_1, \alpha_2) \rightarrow R_+ \cup \{0\}$.

A plane figure bounded by straight lines $\alpha = \alpha_1$, $\alpha = \alpha_2$ $0 \leq \alpha_1 \leq \alpha_2 \leq 2\pi$ bounded with a curve given in the polar coordinates $r = f(\alpha)$, is called a curvilinear sector.

Applet TECOMP-Polarne koordinate (Figure 12.) shows curvilinear sector OBA bounded with the graph of $r = f(\alpha)$, $\alpha \in (0, \alpha)$, and lines $r_1 = f(0), r_3 = f(\alpha)$, which is located between two circular sectors of radius r_1 and r_3, and the central angle α. We observe a circular sector inscribed in a curvilinear sector, of radius r_3 and a circular sector described around it of radius r_1, with the same central angle.

The area of the circular section is calculated as $P_i = \dfrac{r^2}{2}$, where r is the radius and the angle α is given in radians. The area of a curvilinear sector OBA is larger than the area of the inscribed circular sector and smaller than the area of the described circular sector. Next, the angle α is divided into angles $\alpha_i, i = 0,1,2,\dots n$, and $0 = \alpha_0 < \alpha_1 < \cdots < \alpha_n = \alpha,$, which can be the same or different sizes. An inscribed and described circular sector is formed for each angle and the corresponding curvilinear sector. The area of a curvilinear sector is larger than the sum of the areas of the inscribed circular sectors and smaller then the sum of the areas of the described circular sectors. If the function f is increasing, and the division is equidistant then $\Delta\alpha = \alpha/n$, then it holds
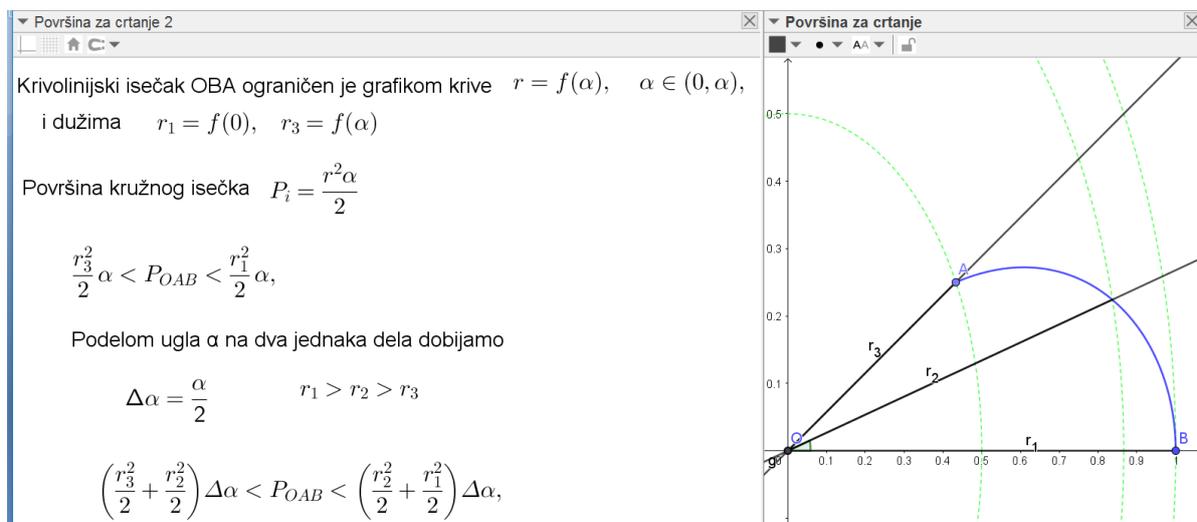
Figure 12.

$$\sum_{i=1}^{n} \frac{f^2(\alpha_{i-1})}{2} \Delta\alpha \ < P_{OBA} < \sum_{i=1}^{n} \frac{f^2(\alpha_i)}{2} \Delta\alpha$$

If the limits of the sums (in upper relation) exist when $n \rightarrow \infty$ ($max_i \Delta\alpha_i \rightarrow 0$, where $\Delta\alpha_i = |\alpha_i - \alpha_{i-1}|, i = 1,2,\dots,n$) then it defines

$$P = \frac{1}{2} \int_0^{\alpha} f^2(\alpha) d\alpha$$

122

which is equal to the area of the corresponding curvilinear sector.

The applet TECOMP-cvet (Figure 13.) shows the curves $r = \cos n\alpha$, za $n = 3$ i $n = 4$. The slider n enable the change of n in order to obtain flowers with different number of lattice. The area of each flower is the same. On the left graphic view the indefinite integral is calculated for the function $h(x) = \cos^2 nx$, multiplied by ½. It can be seen that definite integral on interval $[0, 2\pi]$ is equal to $\pi/2$.
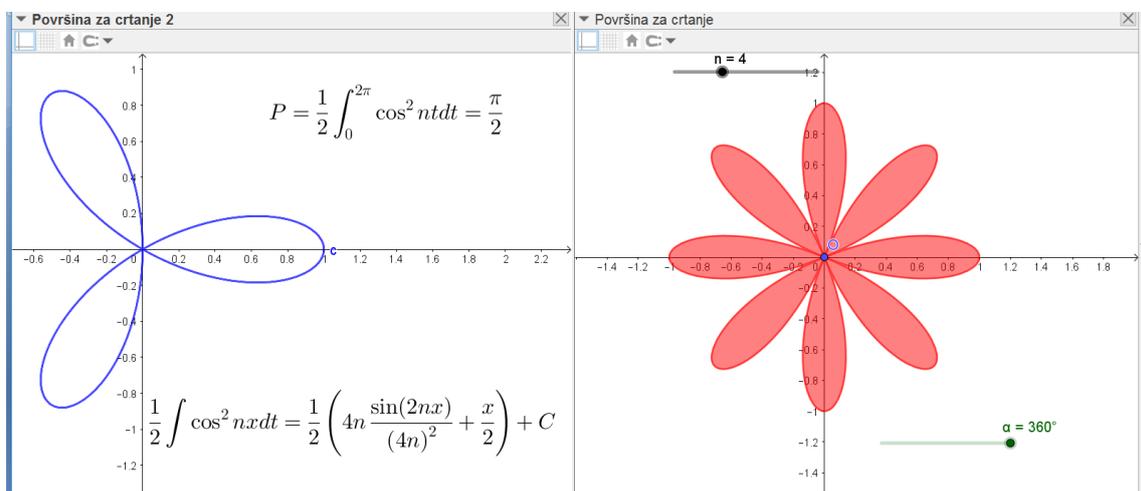
Figure 13.

The curve given in polar coordinates can be inputted in *GeoGebra* by using command

„ Curve(<Expression>, <Expression>, <Parameter Variable>, <Start Value>, <End Value>)",

In particular, for the curve $r = \cos n\alpha$, we input „ Curve ($\cos n\alpha \cos \alpha$, $\sin n\alpha \sin \alpha$, t, 0 , 2π)".

−Volume of solid of revolution

In order better understanding the solids of revolution, we suggest to students to draw a cylinder, a cone and a truncated cone in the *GeoGebra* 3D graphic view, as in Figure 14.a), 14 b), 14 c) A solid of revolution is created by a curve located in a plane, for example xy plane. First, in *Geogebra*, one apply the rotation of the given curve, denoted by f, aroud one of the axes (usually the x −axis) for angle α (given by its slider), making curve f'. After including the trace to f', and applying animation we get the solid of revolution. In Figure 14, each image contains a 2D graphic view as well as 3D graphic view. On 2D graphic view the segments which rotate around x −axis, and form in 3D graphic view the cylinder, the cone and the truncated cone, are shown. Let us remark that the points A 'and B' in all three figures in Figure 14 are formed by rotating segment AB for 180 °.
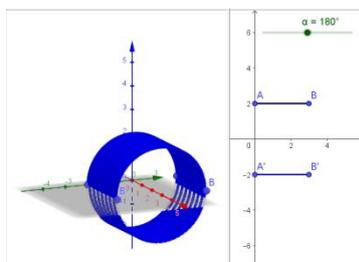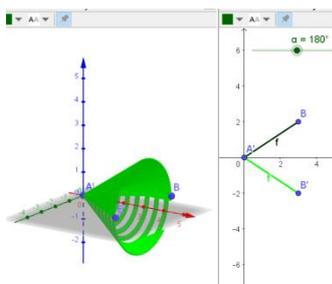
Figure 14 a)   Figure 14 b)   Figure 14 c)
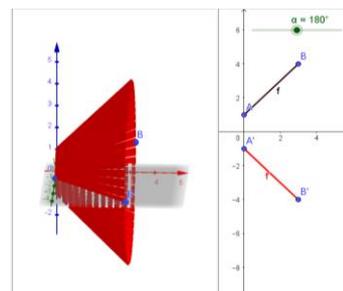
In order to obtain the formula for the volume of revolution of curvilinear trapezoid (determined by continuous function $y = f(x)$ on $[a, b]$ na $x-$ axis), around $x-$ axis, we divide the interval $[a, b]$ on n subintervals $\Delta x_i, i = 1,2, \dots n$, and determine describes and inscribes rectangles, for each subinterval. Each rectangle by rotation of around $x-$ axis, form the cylinder, which height is equal Δx_i, and radijus is equal $f(x_{i,min})$ or $f(x_{i,max})$. The volume of such cilinder je $V_i = \pi f^2(c_i)\Delta x_i$, $c_i \in \Delta x_i, i = 1,2, \dots, n$. If we denote by V the volume of revolution of considered curvilinear trapezoid, then it holds:

$$\pi \sum_{i=1}^{n} f^2(x_{i,min}) \, \Delta \, x_i < V < \pi \sum_{i=1}^{n} f^2(x_{i,max}) \, \Delta \, x_i$$

and

$$\lim_{n\to\infty} \pi \sum_{i=1}^{n} f^2(c_i) \, \Delta \, x_i = \pi \int_{a}^{b} f^2(x)dx = V,$$

if the limits of upper sums exist for each division of the interval $[a, b]$ anf for each choice of n points $c_i \in [x_{i-1}x_i]$. Let us remark that in the last integral subintegral function is positive and the value of definite integral will be positive.

Let us remark that if the curve rotate around $x-$ axis, the intervals on $x-$ osi, are considered, while if the rotation is done around $y-$ axis, then the intervals on $y-$ osi are considered.

The applet TECOM-ZapreminaInt in Figure 15. shows the function $f(x) = 2\sqrt{x}, \ x \in [0,1]$. The graph of f is rotated around $x-$ ose forming solid of revolution, which volume is:

$$V = \pi \int_{0}^{1} 4xdx = 2\pi.$$

The interval $[0,1]$, on $x-$ axis, is divided on 5 subinterval of the same length $\Delta x = 0.2$. The rectangles decribed and inscribed for each subinterval, rotate around $x-$axis and form cilinders. In Figure15. solid of revolution (around $x-$axis) is shown in orange, the decribed cylinders for each subinterval are in blue. It can be shown that the sum of areas of decribed cylinders is:

$$V_o = 4\pi(0.2 + 0.4 + 0.6 + 0.8 + 1)0.2 = 2.4\pi.$$
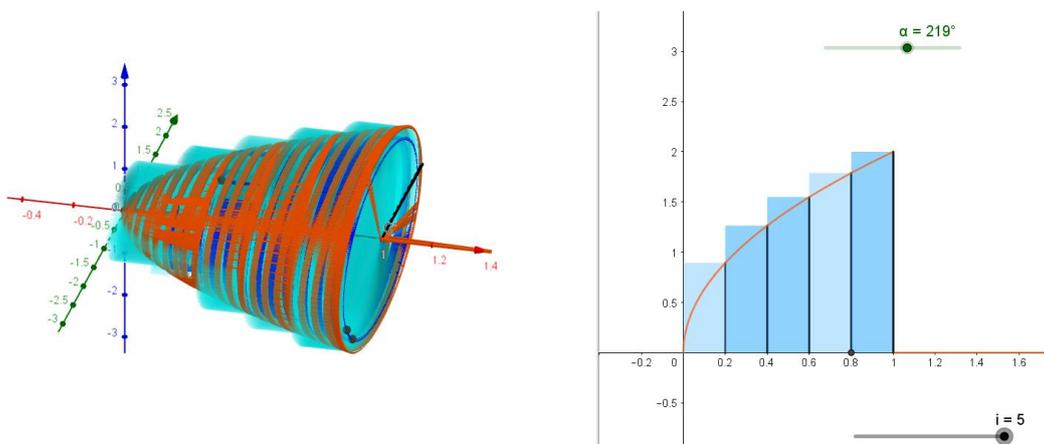
Figure 15.

Inscribed cylinder can be analysed, by using the same applet and determine their sum, too:

$$V_o = 4\pi(0 + 0.2 + 0.4 + 0.6 + 0.8)0.2 = 1.6\pi.$$

If the graph of function $y = f(x)$, $x \in [a, b]$ is rotating around $y-$axis then the volume of such solid of revolution is:

$$V = \pi \int_c^d (f^{-1})^2(y)dy, \qquad c = f(a), \ d = f(b),$$

where f^{-1} is inverse function of f.

The function $f^{-1}(y) = \frac{y^2}{4}, y \in [0,2]$, is inverse to the function $f(x) = 2\sqrt{x}, \ x \in [0,1]$. Then the volume of solid revolution for the function f around oko $y-$axis is:

$$V = \pi \int_c^d (f^{-1})^2(y)dy = \pi \int_0^2 \frac{y^4}{16} dy = \pi \frac{y^5}{80} |_0^2 = \pi \frac{8}{25}.$$
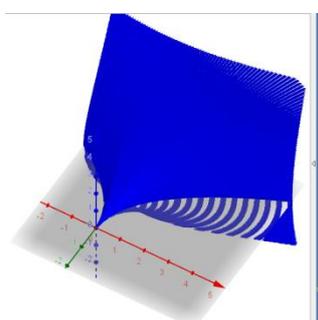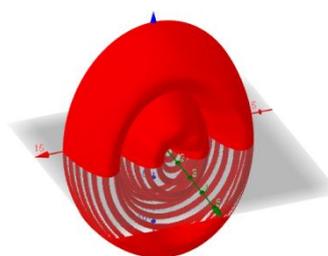
| Figure 16 a) | Figure 16 b) |

In Figures 16 a), 16 b), the solids of revolution of the graph of $f(x) = 2\sqrt{x}$, and cycloid in parametric form aroud $y -$ axis are shown.

The volume of the solid of revolution of cycloide $((t - \sin(t)), (1 - \cos(t)) \ y -$ ose $t \in [0, \pi])$, around $y -$ axis is

$$V = \pi \int_0^\pi (t - \sin t))^2 \sin t \, dt = 2.27.$$

Subintegral function is positive on $[0, \pi]$. The volume of solids of revolution for cycloid is determined on applet Zap-cikloida-y-osa.

−Arc length of a curve

Arc length, s, (Figure 17.) of the graph of continuous function $y = f(x)$ on $[a, b]$ is determined by using chord connecting the points $C(a, f(a))$, $B(b, f(b))$, denoting by $g = BC$. It holds:

$$g^2 = (\Delta x)^2 + (\Delta y)^2 = \left(1 + \frac{(\Delta y)^2}{(\Delta x)^2}\right)(\Delta x)^2, \quad g = \sqrt{\left(1 + \frac{(\Delta y)^2}{(\Delta x)^2}\right)}\Delta x.$$

The interval $[a, b]$ is divided on n subintervals, $\Delta x_i, i = 1,2, \dots n$, and the corresponding subintervals of the values of functions $\Delta y_i, i = 1,2, \dots n$ are obtained. If n tends to infinity and length of biggest interval is tending to 0, then

$$\lim_{n \to \infty} \frac{\Delta y_i}{\Delta x_i} = \lim_{\Delta x \to 0} \frac{\Delta y_i}{\Delta x_i} = f'(x_i), \quad x_i \in \Delta x_i,$$

and

$$\lim_{n\to\infty} \sum_{i=i}^{n} \sqrt{\left(1 + \frac{(\Delta y_i)^2}{(\Delta x_i)^2}\right)} \Delta x_i = \int_a^b \sqrt{1 + (f'(x))^2} dx = s.$$

For example, the length of the graph of function $f(x) = 2\sqrt{x}$, $(f'(x) = \frac{1}{\sqrt{x}})$ on interval $[0.25,4]$ is:

$$s = \int_{.25}^{4} \sqrt{1 + 1/x} dx = 4.88.$$

The previous integral can be solved by using $\sqrt{1 + 1/x} = t^2$, which makes rational subintegral function. The function $f^{-1}(y) = \frac{y^2}{4}$, $y \in [1,4]$ is inverse to the function f, on the interval $[0.25,4]$. The length of the graph of f^{-1}, on interval $[1,4]$, calculating by using $y - $ axis is

$$s = \int_1^4 \sqrt{1 + (f^{-1}(y))')^2} dy = \int_1^4 \sqrt{1 + (y/2)^2} dy = 4.88,$$

the same, no matter along which axis they are calculated.

We suggested determining the area of circle and its length.
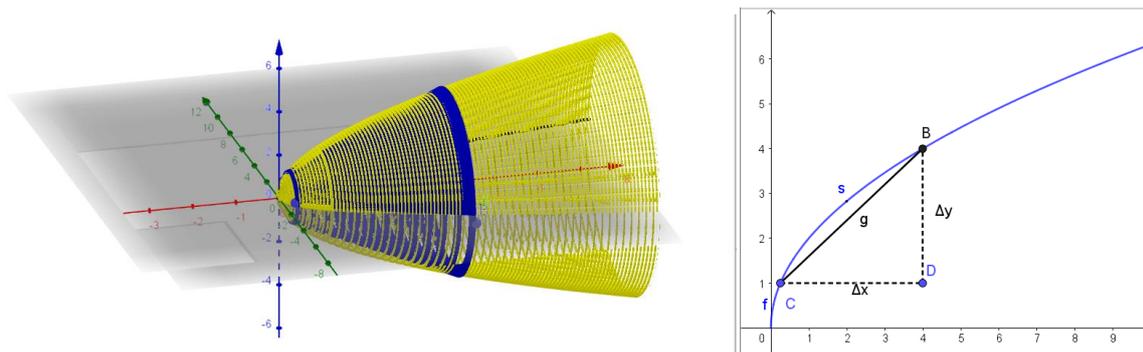
Figure 17.

−Lateral area of the solid of revolution

In order to determine the lateral area of the solid of revolution of the function $y = f(x)$ on the interval $[a, b]$ around of $x - $ axis, we will first consider the solid of revolution of segment connecting two end points of the function f, on the interval $[a, b]$. In Figure 17., this is segment BC, and by its rotation around $x - $ axis one can obtain lateral area of the truncated cone which lateral area is $P = \pi(r_1 + r_2)s$, where r_1, r_2 are radii, and s is the slant height. In this case $r_1 = f(a)$, $r_2 = f(b)$, and s is the length of f.

Applet TECOMP-površinaObrtnih tela shows the solid of revolution of $f(x) = 2\sqrt{x}$, in yellow color and inside it ca be seen the truncated cone. The graph in 3D se can be rotated.

The interval $[a, b]$ is divided on n subintervals $\Delta x_i, i = 1, 2, \ldots n$, and the graph of f together with the segment connecting the end points of the graph is rotated on each subinterval. So, the sequence of the sums of lateral areas of the truncated cones is obtained, which limit, if exists regardless of the

division, and if the length of the biggest interval is tending to zero, is equal to the lateral areal of the solid of revolution:

$$\lim_{n \to \infty} \pi \sum_{i=i}^{n} (f(x_{i-1}) + f(x_i)) \sqrt{\left(1 + \frac{(\Delta y_i)^2}{(\Delta x_i)^2}\right)} \Delta x_i = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2} dx = P_0$$

If the graph of $f(x) = 2\sqrt{x}$, $x \in [0.25, 4]$, is rotated aroud $x - $ axis the solid revolution is obtained with the lateral area:

$$P_0 = 2\pi \int_{0.25}^4 2\sqrt{x}\sqrt{1 + \frac{1}{x}} dx = 4\pi \int_{.25}^4 \sqrt{1 + x} dx = 4\pi \frac{2(1 + x)^{\frac{3}{2}}}{3}\Big|_{0.25}^4 = 26.8\pi.$$

The lateral area of the solid of revolution of the same function around $y - $ axis is obtained as: (the formula can be derived analogously).

$$P_0 = 2\pi \int_a^b f^{-1}(y)\sqrt{1 + ((f^{-1})'(y))^2} dx = 2\pi \int_1^4 \frac{y^2}{4}\sqrt{1 + \frac{y^2}{4}} dy = 19{,}22\pi.$$

The last integral can be solved by using Ostrogradski method, and by using *GeoGebra*, too.

Applet TECOMP-Torus (Figure18.) shows the torus determined by the rotation of $x^2 + (y - a)^2 = r^2$ around $x - $ axis. The volume of torus can be obtain as the difference between two volumes of solid of revolution determined by the curves $y_1 = a + \sqrt{r^2 - x^2}$ i $y_2 = a - \sqrt{r^2 - x^2}$

$$V = \pi \int_{-r}^r (y_1^2 - y_2^2) dx = 4a\pi \int_{-r}^r \sqrt{r^2 - x^2}\, dx = 2ar^2\pi^2.$$

The area of torus is the sum of two areas determine by curves y_1, and y_2. From $y'_{1,2} = \pm \frac{x}{\sqrt{r^2 - x^2}}$ it follows

$$P = 2\pi \int_{-r}^r (y_1^2\sqrt{1 + (y_1')^2} + y_2^2\sqrt{1 + (y_2')^2}) dx = 4\pi^2 r.$$
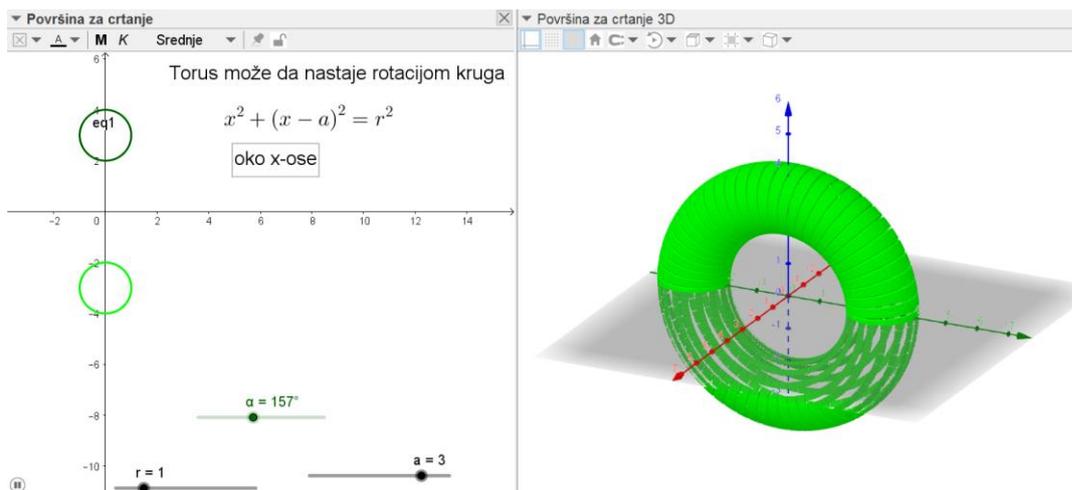
Figure 18.

The applet TECOMP-EliHip shows the solids of revolution (elipsoid and hiperboloid) in 3D graphic view derived by curve $ax^2 + by^2 = ab$, depending on the parameters a i b. If $a = b$, then the sphere is obtained, (applet TECOMP-sphere).

References

Anthony, B., Jr, Kamaludin, A., Romli, A., Raffei, A., F., M., Eh Phon, D., N., A., L., Abdullah, A., Ming, G., L., (2020), Blended Learning Adoption and Implementation in Higher Education: A Theoretical and Systematic Review, Technology, Knowledge and Learning, https://doi.org/10.1007/s10758-020-09477-z

Graham, C. R. (2013). Emerging practice and research in blended learning. *Handbook of Distance Education,3*, 333–350.

Hadžić, O., Takači, Đ., (1998), Matematika za studente prirodnih nauka, Univerzitet u Novom Sadu, Novi Sad.

Hadžić, O., Takači, Đ., (2000), Matematičke metode, Univerzitet u Novom Sadu, Novi Sad.

Mortenson, M., (2020), Vectors and Matrices for Geometric and 3D Modeling, Industrial press inc. http://www.springerlink.com/index/N423R5P1011388J1.pdf

Sarah Guri-Rosenblit, Distance education' and 'e-learning': Not the same thing, (2005), Higher Education 49: 467–493

Radenović, S., Takači, Đ., (2002), Matematika 1- za inženjere, Akademska misao, Beograd.

Abrahamson, C. E. (1998). Issues in interactive communication in distance education. *College Student Journal, 32*(1), 33 – 43.

Laurillard, D. (1993) Rethinking University Teaching: A Framework for the Effective Use of Educational Technology, Routledge/Falmer, London.

Author:
Sana Stojanović Đurđević

QUESTIONS:

What is interactive theorem proving?

Why Isabelle?

Mathematical logic and Isabelle?

Programming and Isabelle?

Theorem prover Isabelle in the teaching of natural and mathematical sciences

The shortest path from mathematical logic to formally verified and more efficient programs

Contents

Introduction to interactive theorem proving

The material in front of you is based on the book "Uvod u interaktivno dokazivanje teorema" (Introduction to interactive theorem proving), written by the author of this training material and professor Filip Marić. Course with this subject is taught in the IV year at the Faculty of Mathematics, University of Belgrade. The course started in the school year 2018/2019.

Although there are many programs for interactive theorem proving, currently we are working only with interactive theorem prover Isabelle/HOL. One of the reasons for this choice is

Isabelle's very rich notation, syntax and declarative language which can be used for writing evidence. This makes it somewhat close to classical mathematics, which is close to both students and mathematicians and programmers. In addition, Isabelle/HOL offers extensive support for functional programming and formal software verification.

Whole material is created and tested directly in the interactive prover Isabelle. We are using Isabelle/HOL theories, which contain definitions, lemmas and their proofs. Each theorie can be loaded and run individually, ie. checked within the prover Isabelle. This approach to creating material was much more demanding for the authors and special attention was paid to aligning the code translated in Isabelle with the text describing its meaning. This type of organization offers the reader a unique opportunity to test the existing material (without individually typing and extracting parts of the code that interests him), to learn directly from it, to supplement, change and master it step by step.

The current version of the book is available at: http://poincare.matf.bg.ac.rs/~sana//uidt/uidt.pdf

We hope that this will provide access to a wider variety of readers and eventually lead to greater popularity of the use of computers for the purpose of proving theorems and formal verification of software.

Electronic materials, which can be used for the user to independently generate a PDF version of the book based on the original Isabelle/HOL code, are located at the same address. Electronic material is divided by chapters and is available in both Isabelle *thy* format, and in *htm* format. For that reason, a small part of the book is dedicated to creating output documents based on pre-created theory.

Theorem proving with a computer

Computers were initially used for *automated theorem proving* only. Mathematical theorems were formulated within an automated theorem prover, and the prover alone was used to *prove or disprove* that theorem (within a fixed formal system). The output was, in most cases, yes/no answer without much additional information about a proof. Today, automated theorem proving is still widely used with many specific mathematical theories (hardware verification, geometry, etc.).

But, automated theorem proving has limited success with difficult mathematical theorems. Automated theorem provers are mostly used with a lot of help from a human (mathematician). In the last few decades, *interactive theorem proving* has gained new attention. A user (mathematician) is using a computer to complete parts of its proof, or to write the whole proof completely on its own. Interaction between a computer and a human is much greater than with automated theorem provers. Computers are mainly used to formally verify given proof (although they can still be used to automatically generate small parts of the proof).

Some of the most frequently used interactive theorem provers today are:
- Isabelle (http://www.cl.cam.ac.uk/research/hvg/Isabelle/)
- Coq (http://coq.inria.fr/)
- Lean (https://leanprover.github.io/)

- Mizar (http://www.mizar.org/)
- HOL-Light (http://www.cl.cam.ac.uk/~jrh13/hol-light/)

Today, it is usual to use collaboration between traditional automatic and interactive proofs, and interactive proofs have the ability to use some automatic provers. Although automatic provers usually do not generate complete formal proof, they can provide information on a set of axioms and auxiliary lemmas used during the proof, which the interactive prover can use to reconstruct and verify the entire formal proof.

Interactive theorem provers are considered much more reliable than traditional automatic provers, because they insist that each proof be derived in full detail in a given formal system. In the final instance (which can be hidden from the user), each piece of proof is expressed in the form of the application of basic axioms and rules of derivation and the so-called *trusted core* checks the correctness of each piece of proof. Proof is accepted only if it is successfully verified by a trusted core. Note that all advanced components of the interactive theorem prover that can find parts of proof do not have to be formally verified, because if they contain an error and find a proof that is incorrect, the trusted core of Isabelle will detect it and such proof will be discarded.

The discovery of errors in mathematical proofs, in textbooks and published papers, and in programs used in life-critical situations, has led to the need for machine-verifiable proofs of theorems and formal verification of written programs. In addition to their application in education and industry, interactive theorem provers significantly contribute to the preservation of a rich mathematical heritage through the formalization of important mathematical books.

The two most commonly used approaches in the interactive theorem proving are the procedural and the declarative approach. *The procedural approach* makes extensive use of automated tools or tactics with the task of simplifying the current objective of the theorem. By carefully combining tactics, the goal of the theorem is simplified until the assumptions of the theorem are reached, when we can say that the proof is complete. Thus, in this approach, the proof consists of a series of calls of tactics that transform the theorem being proved and reduce it to simpler statements. When using *the declarative approach*, the proof is written in a language reminiscent of the standard text that appears in mathematical textbooks, thanks to which proofs are similar to the proofs from mathematical textbooks both in structure and syntax. Declarative evidence is therefore more accessible to the average mathematician and student, as opposed to procedural evidence which requires a little more knowledge of the language over which it is written.

History of Isabelle

Interactive theorem prover Isabelle has been used since the mid-1980s. The most important authors, with the most contributions, are Larry Paulson from the University of Cambridge and Tobias Nipkow and Markus Wenzel from the Technical University of Munich. Theorem prover Isabelle is to some extent based on the LCF prover and can be said to follow the LCF approach to ensure the correctness of the system (it implies the existence of a verified core and the specific use of a functional programming language that provides reliable verification of proof without the need to explicitly construct and store whole proof in a computer memory).

Theorem prover Isabelle is a generic prover and allows working with various formal systems (so-called object logics). Within Isabelle, a *meta-logic* called Isabelle/Pure is implemented, within which various object logics are defined. Isabelle supports several object logics, the most important of which are:

- HOL - higher order logic,
- FOL - classical and intuitionistic first order logic,
- ZF - Zermelo-Fraenkel set theory,
- CTT - a version of constructive type theory,
- LCF - a version of Scott's logic for computable functions (build upon classical FOL),
- HOLCF - a version of LCF, defined as an extension of HOL.

The object logic of the Isabelle system, which is by far the most widely used, is *Higher Order Logic*. The version of Isabelle's prover that uses higher-order logic is called Isabelle/HOL and it appeared in the early 1990s. This version will be represented in this book. The higher-order logic in the Isabelle/HOL prover is based on a typed version of the lambda-calculus, but in this book we will not deal with the logical foundations of the system, but only with its practical application.

In addition to proving mathematical theorems and formalizing mathematical theories, Isabelle/HOL is also used extensively as a program verification tool. Namely, Isabelle/HOL offers a functional programming language in which it is possible to define functions that implement different algorithms, and then formulate and prove their properties. Based on these definitions, it is possible to automatically export program code in several functional programming languages (Haskell, SML, OCaml).

Installation and use of Isabelle/HOL

Interactive theorem prover Isabelle/HOL can be downloaded from the following page https://isabelle.in.tum.de/. The version used for this material is Isabelle2021/HOL. Mainly, installation is trivial, but there are additional instructions on the following page https://isabelle.in.tum.de/installation.html.
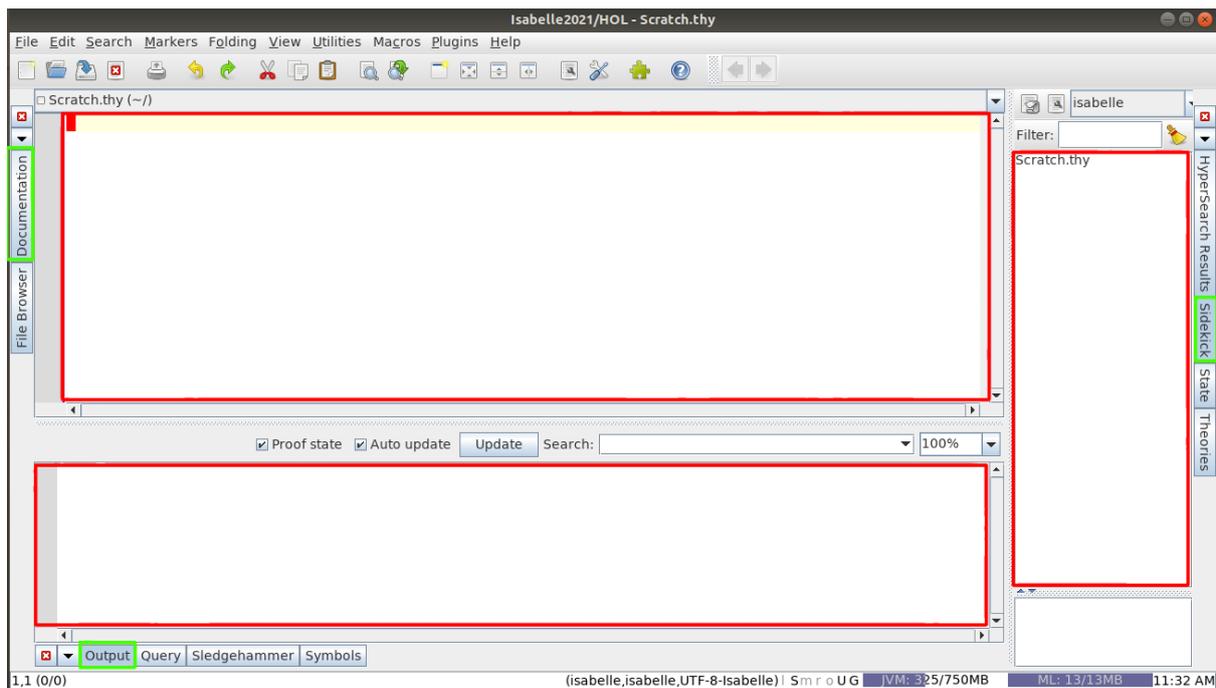
Picture 1. Windows theory, content and output are marked in red. Buttons that can be used are marked in green.

After installation of Isabelle/HOL, three main frames are visible: *theory* (upper half), *content* (far right), i *output* (lower half). If the output box is not open, press the button **Output** (bottom of the screen). If the content box is not open, press the button **Sidekick** (on the right).

Additionally, the rich Isabelle / HOL documentation can be accessed by pressing the **Documentation** button on the left side of the screen. This documentation can also be accessed online from

http://isabelle.in.tum.de/documentation.html, as well as from the folder in which Isabelle/HOL is installed (in subfolder *doc*).

All theories, definitions and lemmas that are proven are entered into the *theory* frame. The *output* frame displays all current goals, and current state for each step of the proof. The *content* frame shows the current structure of the document from the *theory* frame.

An example of Isabelle/HOL theory

The following simple example illustrates the basic concepts of the Isabelle system. The basic unit of content in Isabelle/HOL is a *theory*. Within a theory, definitions are introduced and claims are formulated and proved. Although in mathematics we distinguish between lemmas and theorems, in the formal system there is no difference between them (all of these are statements that are formulated, proved and used later in the same way). Consider a very simple example of a theory.

```
theory follower
  imports Main
begin

definition follower :: "nat \<Rightarrow> nat" where
  "follower x = x + 1"
```

```
lemma "follower (follower x) = x + 2"
  unfolding follower_def
  by auto

lemma
  assumes "x > 0 \<and> x < 3"
  shows "follower x > 1 \<and> follower x < 4"
  using assms
  unfolding follower_def
  by auto

end
```

After copying this text into Isabelle, saving it with a name "follower.thy" and moving the cursor through the theory, we can see how Isabelle/HOL interprets each line of the proof (the line on which the cursor is currently located).

In order to follow the interpretation of each command in Isabelle, it is necessary to select the **Theories** button on the right side, in the *overview* frame, and select the **Continuous checking** option. All currently open options are displayed in this window.

Additionally, in order to see the video output in the Isabelle/HOL system, it is necessary to open the *Output* frame at the bottom of the screen and check the **Proof state** option.

Explanation of the example:

At the beginning, the function *follower* is defined. Isabelle/HOL is a typed language and all objects have their own type. The type of this function is *"nat => nat"*, which means that this is a function that maps natural numbers into natural numbers. After specifying the type, an equation is given which defines the function *follower*. Note that, in accordance with the practice of functional programming, the call, i.e. the application of the function is written without parentheses (instead of *follower*(x), *follower* x is written). More words on syntax will be given later in this chapter.

In the following, two lemmas are formulated and proved. The formulation of the first lemma is obvious. It is proved completely by using an automatic prover (which is called the words **by auto**). The **unfolding follower_def** command is used to apply the definition of the *follower* function before applying the automatic prover. This needs to be stated individually, because automatic provers usually do not use definitions (otherwise the theorem that is proved would very quickly lead to the most basic concepts on which the whole formal system is based and the theorem would become too cumbersome and complicated).

And the next lemma is very obvious. The construction **assumes - shows** formulates the assumption and conclusion of the lemma. The first lemma had only a conclusion, so it was possible to omit the keyword **shows**, although it would not be a mistake even if it was listed. Within the assumption and within the conclusion, the conjunction was used (how mathematical symbols are entered using the

keyboard will be discussed below). The command **using assms**, is emphasizing to the automatic prover that when proving a lemma, it should also use an assumption (the default behavior of automatic provers is that they do not try to use assumptions until the user explicitly states them).

44 Isabelle/HOL language syntax

In the Isabelle/HOL system, we distinguish between *internal* and *external* syntax. The fixed syntax used to define the theory (of which the keywords *begin* and *datatype* are part) is called *external syntax*. *Internal syntax* is defined by the language of the theory that the user wants to create (description of the theory, its definitions, lemmas and their evidence).

External syntax defines the syntax used when writing all commands. It consists of keywords like *lemma, datatype, assumes, proof, qed, show, have,* etc. and describes constructs that are allowed within a single theory. Its detailed description can be found in the tutorial "isabelle-isar-ref.pdf". In addition, it consists of type names (*float, int, nat*), `special mathematical symbols such as` ∀ `and` λ (`which are written in Isabelle/HOL in the usual way` (\\<*forall*> and \\<*lambda*>), commands for writing text (written under the *text* command), invisible comments (written under the *(* *)* command), and visible comments (written under the \\<*comment*> command).

Internal syntax refers to text that is enclosed in double quotes. For example, when defining a specific notation that is used in the desired logical framework that the user develops (after keywords such as *definition, example, datatype*).

When writing *internal syntax*, the use of quotation marks is necessary, except in the case of individual variables (for example, "x + y" and z are allowed, but x + y is not written without quotation marks). The default behavior is that these quotes are not visible when generating pdf documents.

When reporting a syntax error, Isabelle will print an *Inner syntax error* for an error that occurs within the internal syntax, or an *Outer syntax error* for an error that occurs within the external syntax.

Basic terms

This chapter will present the basic mathematical concepts and syntax used to write them in the Isabelle/HOL language. The language used was created in the spirit of a long tradition of functional programming. In the introductory example, we saw that Isabelle/HOL is a typed language (we used the natural number type *nat*).

The basic data types in the Isabelle/HOL language are: *bool, nat, int, real, complex* correspond in order: to the type of logical values, to the type of natural, integer real and complex numbers.

Derived data types are created with the help of so-called. type constructors: for example, a *list* for lists and a *set* for creating sets of elements of a given type. These two constructors are written postfixed, e.g. a list of natural numbers is written as a *nat list*, and a set of real numbers as a *real set*.

The type of functions is specified by the operator ⇒. For example, the type *nat ⇒ real* denotes a series of real numbers (a function that assigns a real value to each index that is a natural number). In a situation where the function has more than one input parameter, the curry form of the function is used and it is written in the form *nat ⇒ nat ⇒ real*. This form of the function is more flexible and has many advantages over the notation *nat × nat ⇒ real* which uses torques, of which perhaps the most important

is that it works well with the principle of mathematical induction. In Isabelle/HOL (similar to Haskell) functions are always defined as Curry functions (except where torques are not explicitly required).

Type variables - are used to write polymorphic functions, i.e. when we need to specify a variable of arbitrary type instead of a specific type. Typical variables are specified with an apostrophe in front of: *'a,' b*. Many standard functions are defined as polymorphic. For example, a function that calculates the length of a list has type *'a list ⇒ nat*, which means that its argument is a list of elements of any type, and its result is a natural number.

Terms are created as in functional programming languages. A function *f* of type *T1 ⇒ T2* applied to a term *t* of type *T1* will create a term *f t* of type *T2*. If we want to emphasize that the term *t* will be of type *T* then we write *t :: T*. Isabelle/HOL also supports lambda functions, the use of which will be illustrated in later chapters. The use of constants (function of arity 0) and variables as base terms is allowed.

Isabelle/HOL also supports some of the basic constructs taken from functional programming languages (these three constructs are always listed between parentheses). *let-expression* introduces values to variables, *case-expression* defines a value depending on the form of the expression, *if-expression* is a conditional expression that stands for case expression.

(let x1 = t1 x2 = t2 in u)

(case t of pat1 ⇒ t1 |… | patn ⇒ tn)

(if b then t1 else t2)

Writing formulas in Isabelle/HOL

This chapter will show the basic Isabelle/HOL syntax used to write formulas. In the beginning, we will not look at the proofs of the given claims, but we will rely on automatic proofs.

Formulas are expressions over the type *bool* with the standard constants *True* and *False* and with the usual logical conjunctions (listed in ascending order of priority): ¬, ∧, ∨, →. The conjunction → has a right associativity, which means that the next expression A → B → C has the meaning as if the brackets are placed like this A → (B → C), while the other binary conjunctions have a left associativity.

Equality is usually used as an infix operator (*=)* whose type is `a ⇒` a ⇒ *bool,* which means that it can be applied to objects of different types (but mutually equal), and even over formulas. Equality of formulas has the meaning "if and only if", and can be written with ↔ (operators = and ↔ have the same logical meaning when applied to logical formulas, i.e. bool-type terms, but operator ↔ has much lower priority).

The universal and existential quantifiers are written in the usual way ∀ x. P and ∃ x. P.

Formulas of this form, in which conjunctions are applied only to variables of type *bool* and in which quantifiers are not used, will be called *statement formulas*. Formulas in which quantification is performed only on variables of elementary types (e.g. over numbers) will be called *first-order logic formulas*, and formulas in which quantification is performed on functions will be called *higher-order logic formulas*. `For example, the formula ¬ (A ∧ B) ↔ A ∨ B is a statement formula, the formula ¬ (∀ x. P x) ↔ (∃ x. ¬ P x) is a first−order logic formula, while the formula (∀ x. ∃ y. P xy) ↔ (∃ f. ∀ x. P x (fx)) higher order logic formula.`

The most commonly used logical symbols and their ASCII notation (in extended and abbreviated form) are listed below. These (and similar) symbols can be printed as a command or loaded when typing a command (by pressing the *Tab* button when the system recognizes and offers several possible symbols) or, in situations when a previously written Isabelle/HOL file is loaded, by selecting a menu option *File / Reload with Encoding / UTF-8-Isabelle* which will reload symbols instead of ASCII symbol entries.

∀	\<forall>	ALL
∃	\<exists>	EX
λ	\<lambda>	%
→	-->	
↔	<->	
∧	/\	&
∨	\/	|
¬	\<not>	~
≠	\<noteq>	~=

For all the listed conjunctions, in Isabelle/HOL we can use the *term* command to get the type of conjunction. When testing a type, the conjunction must be specified in parentheses, e.g. *term (^).*

When formulating theories, definitions and statements are written using the syntax described so far. This syntax represents the syntax of the object level, and for the formulas we have seen so far we say that the formulas of the object level are in higher order Isabelle/HOL logic. Internally, however, this object logic is just one of the various logics that the Isabelle system knows. All these logics (including this Isabelle/HOL logic) are defined within the meta-logic of the Isabelle system.

45 Example of exam

Below are presented a few types of problems that can be used to review basic examples that can be easily demonstrated in Isabelle. The formulation of the problem is given first, and then its solution is presented. All problems can be independently checked in Isabelle.

```
theory Exam
imports Complex_Main
```

begin

(* 1 *)

text \<open>Prove the following lemmas by using the rules of natural deduction for classical first-order logic.

Note: The proof of the first lemma is part of the proof of the second lemma. In case you fail to prove the second proof to the end, try the first proof to get partial points.\<close>

(* 4 points – if only this lemma is done *)
(* if you prove the next lemma, you don't have to do this one *)

lemma "(\<forall> x. (G x \<longrightarrow> \<not> I x) \<longrightarrow> F x \<and> H x) \<and> (\<forall> x. \<not> H x) \<longrightarrow> (\<forall> x. I x)"
  apply (rule impI)
  apply (erule conjE)
  apply (rule allI)
  apply (erule_tac x = x in allE)
  apply (erule_tac x = x in allE)
  apply (rule ccontr)
  apply (erule impE)
   apply (rule impI)
   apply assumption
  apply (erule conjE)
  apply (erule notE)
  apply assumption
  done

(* 10 points *)
lemma "(\<forall> x. F x \<or> G x \<longrightarrow> \<not> H x) \<and> (\<forall> x. (G x \<longrightarrow> \<not> I x) \<longrightarrow> F x \<and> H x) \<longrightarrow> (\<forall> x. I x)"
  apply (rule impI)
  apply (erule conjE)
  apply (rule allI)
  apply (erule_tac x = x in allE)
  apply (erule_tac x = x in allE)
  apply (erule impE)
  apply (rule ccontr)
  apply (erule impE)
   apply (rule impI)
   apply (erule notE)
   apply (rule disjI2)
   apply assumption
  apply (erule notE)
   apply (erule conjE)
  apply (rule disjI1)
  apply assumption
  apply (rule ccontr)
  apply (erule impE)
   apply (rule impI)
   apply assumption
  apply (erule conjE)

```
  apply (erule notE)
  apply assumption
  done


(* 2 *)

(* Prove that the following lemma is valid in Isar. Provide detailed proof *)

(* Note: if you fail to prove equivalence, divide the lemma into two lemmas for subset and superset
*)
lemma
  assumes "inj f"
  shows "f ` (A \<inter> B) = f ` A \<inter> f ` B"
  (is "?l = ?d")
proof
  show "?l \<subseteq> ?d"
  proof
    fix y
    assume "y \<in> ?l"
    from this obtain x where "x \<in> A \<inter> B" "f x = y" by auto
    from this have "x \<in> A" "x \<in> B" by auto
    from this `f x = y` have "y \<in> f ` A" "y \<in> f ` B" by auto
    from this show "y \<in> ?d" by auto
  qed
next
  show "?d \<subseteq> ?l"
  proof
    fix y
    assume "y \<in> ?d"
    from this have "y \<in> f ` A" "y \<in> f ` B" by auto
    from this obtain xa where "xa \<in> A" "f xa = y" by auto
    from \<open>y \<in> f ` B\<close> obtain xb where "xb \<in> B" "f xb = y" by auto
    from this `f xa = y` have "f xa = f xb" by auto
    from this `inj f` have "xa = xb" by (auto simp add: inj_def)
    from this `xa \<in> A` `xb \<in> B` have "xa \<in> A \<inter> B" by auto
    from this `f xa = y` show "y \<in> f ` (A \<inter> B)" by auto
  qed
qed


(* 3 *)

(* Using only the following theorems, prove the next equation step by step *)
(* Note: only use command "simp only", eventually command "simp add" *)

thm mult.commute
thm algebra_simps(19)
thm HOL.no_atp(121)
thm semiring_normalization_rules(29)
thm algebra_simps(20)

lemma ab:
  fixes a b ::real
```

```
  shows "(a-b)^2 = a^2-2*a*b+b^2"
proof-
 have "(a-b)^2=(a-b)*(a-b)"
  by (simp only: semiring_normalization_rules(29))
 also have "... = a*(a-b)- (b*(a-b))"
  by (simp only: Groups.algebra_simps(20))
 also have "... = a*a - a*b -(b*a - b*b)"
  by(simp only: Groups.algebra_simps(19))
 also have "... = a^2 - a*b - b*a +b^2"
  by (simp only: semiring_normalization_rules(29))
 also have "... = a^2-2*a*b+b^2"
  by (simp add: mult.commute)
 finally
 show ?thesis
  by auto
qed

(* 4 *)

text\<open> Formalize the next proof (use the rule nat_induct_at_least, with cases base and Suc).
Additionally, the proof can use simp. If must, you can use auto, but some points may be lost).

  Theorem: Prove that for n \<ge> 1, n < 2^n holds.

  Induction by n.
  Base, n = 1. Then it holds: 1 < 2^1 = 2.
  Let k \<ge> 1. Assume that k < 2^k.
  Prove  k + 1 < 2 ^ (k + 1).
  We can prove that
   k + 1 \<le> k + k
  Based on the induction hypothesis we have  k + k < 2^k + 2^k = 2^(k+1).
  Therefore it holds: k + 1 < 2 ^ (k + 1).
\<close>

lemma
 fixes n :: nat
 assumes "n \<ge> 1"
 shows  "n < 2 ^ n"
 using assms
proof (induction n rule: nat_induct_at_least)
 case base
 then show ?case
  by simp
next
 case (Suc k)
 then have "k + 1 \<le> k + k"
  by simp
 also have "... < 2^k + 2^k"
  using Suc(2)
  by simp
 also have "... = 2 ^ (k + 1)"
  by simp
 finally show ?case
  by simp
```

qed

(* 5 *)

(* We will use the following definition for example: *)
definition par1:: "(nat \<times> nat) list"
  where "par1 = [(1,10), (2,20), (3,30)]"

(* Using primitive recursion, define a function based on a list of pairs, that creates a pair of lists (so that the first list contains the first and the second list the second element of individual pairs). *)

primrec rastavi :: "(nat \<times> nat) list \<Rightarrow> nat list \<times> nat list" where
"rastavi [] = ([], [])" |
"rastavi (x # xs) = (let (a, b) = x; (al, bl) = rastavi xs in (a # al, b # bl))"

value "par1"
value "rastavi par1"
(* Expected result ([1, 2, 3], [10, 20, 30]) *)

(* Express this function via library functions, using the functions fst and snd and prove equivalence with the previous definition *)

term fst
term snd

lemma "rastavi xs = (map fst xs, map snd xs)"
  by (induction xs, auto)

(* Define the inverse function that makes a list of pairs from two lists *)
fun sastavi :: "nat list \<Rightarrow> nat list \<Rightarrow> (nat \<times> nat) list" where
   "sastavi [] [] = []"
| "sastavi xs [] = []"
| "sastavi [] ys = []"
| "sastavi (x # xs) (y # ys) = (x, y) # sastavi xs ys"

(* Test with: *)
value "sastavi [1::nat,2,3] [10::nat,20,30]"
value "sastavi [1::nat,2,3] [10::nat,20,30,40,50]"
(* Expected result: [(1, 10), (2, 20), (3, 30)] *)

(* Express this function via library functions, using the zip and prove equivalence with the previous definition *)
term zip

lemma "sastavi xs ys = zip xs ys"
  by (induction xs ys rule: sastavi.induct) auto

end

University of Niš
The TeComp Consortium

www.tecomp.ni.ac.rs
e-mail:
tecomp@ni.ac.rs
tecomp.p2018@gmail.com